# INV 201 Model Solutions
# March 2026

## 1.

**Learning Objectives:**

1. The candidate will understand key types of derivatives.

3. The candidate will understand various applications and risks of derivatives.

**Learning Outcomes:**

(1c) Be able to compare European, American, Bermudan, Asian options, and various exotic options

(1d) Understand the mechanics of derivatives trading including:
- Exchange traded vs OTC
- Central Clearing
- Daily margin variation and daily settle

(3a) Understand the Greeks of derivatives

**Sources:**

Hull-Options Futures and other Derivatives-11th Edition, Chapter 1, p. 34-36

Hull-Options Futures and other Derivatives-11th Edition, Chapter 12, p. 274-276, 281-282

Hull-Options Futures and other Derivatives-11th Edition, Chapter 17, p.384-386, 389-390
Hull-Options Futures and other Derivatives-11th Edition, Chapter 19, p.434-436

**Commentary on Question:**

*Overall, students showed a reasonable understanding of basic option hedging concepts, and many were able to identify the correct direction of hedge outcomes and costs. However, a significant number of answers lacked precision and completeness, particularly in specifying hedge details and providing clear financial reasoning.*

*Performance was generally weaker on questions involving beta-adjusted hedging, American versus European option pricing, and spread strategies, where deeper conceptual understanding was required. Stronger candidates distinguished themselves through accurate calculations, complete answers, and well-justified explanations.*

## 1. Continued

**Solution:**

(a) Construct an options strategy that will achieve the portfolio manager's objective.

*Answer:*
*A $100 million portfolio consists of 20,000 shares of the index with value of 5000. If the portfolio value decreases to $92 million and beta is 1, that represents a 8% decrease in both the portfolio and the index, corresponding to an index value of 4600 (5000 * .92).*
*Hedge strategy should be to purchase an European put options with strike of 4600 expiring in 1-year on 20,000 shares.*

*Comments:*
*Most students performed well, though some overlooked the need to specify that the term was one year.*

(b)

(i) Construct an options strategy if the beta of the portfolio increases from 1.0 to 2.0.

*Answer*
*First, if the portfolio beta is 2, then the number of shares of the index required to be hedged increases to:*

$$2 * \frac{\$100\ million}{5000} = 40{,}000\ shares$$

*Next, calculate the corresponding index return associated with a 8% decrease in portfolio value if beta is 2:*

$$-8\% = 2 * (r_m - 2\%) + 2\%$$
$$r_m = -3\%$$

*The index value associated with a -3% return is $4850.*
*Therefore, hedge strategy should be to purchase an European put options with strike of 4850 expiring in 1-year on 40,000 shares.*

*Comments:*
*The majority of students were unable to correctly calculate the strike price of 4850. A few also failed to recognize that the position should be doubled to 40,000 shares.*

(ii) Explain whether the cost of hedging will increase, decrease or no change under this new strategy.

*Answer:*
*The cost of hedging will increase, since both the hedge notional and the strike price of the put option increased due to the increased beta. This increases the put premium needed to purchase the option.*

# 1. Continued

*Comments:*
*Many students noted that the cost of the hedge increases, but only a small number identified both underlying reasons: (1) the strike price of the put option rises, and (2) the number of puts required increases.*

(c) Identify two issues in the American option prices provided in the table.

*Answer:*
- *American options should be worth at least the price of the otherwise identical European options. From the table, the American put option with strike price 5000 has a price of 320.79 < 322.90 which is the price of the corresponding European put option. This is incorrect.*
- *The second issue is with the price of the American call option with strike price 4800, which is 581.62 in the price table. The correct price should be 579.37 which is the same as the price of the corresponding European call. This is because for a non-dividend-paying stock, it is never optimal to exercise an American call option early, making its price equal to that of an otherwise identical European call.*

*Comments:*
*Some students successfully identified and clearly explained two issues. Others managed to highlight only one, or provided explanations that were incomplete.*

(d) Using European and American option(s):

(i) Recommend the most cost-efficient hedge strategy if the view of economist A is accepted.

(ii) Recommend the most cost-efficient hedge strategy if the view of economist B is accepted.

*Answer:*
*i) Under the view that the index being above 4200 at the end of the year, the most cost-efficient hedge using the given options is via a bear spread of European puts, consisting of: (a) buying 20,000 European index put options maturing in 1 year with strike price 4600, and (b) selling 20,000 European index put options maturing in 1 year with strike price 4200.*
*ii) Under the view that the index staying above 4200 throughout the year, the most cost-efficient hedge using the given options is via a bear spread of European and American puts, consisting of: (a) buying 20,000 European index put options maturing in 1 year with strike price 4600, and (b) selling 20,000 American index put options maturing in 1 year with strike price 4200.*

*Comments:*
*Very few students recognized that a bear spread was required in the first scenario, and even fewer understood that an American put was needed in the second.*

# 1. Continued

(e) Assess whether the cost of hedging would increase, decrease, or not change under an instantaneous increase in the volatility of the index.

*Answer:*
*Option Vega is positive for both calls and puts, hence an increase in volatility increases the prices of the European puts used in the hedge. Holding all else the same, Vega is higher for closer to the money options. So the cost of the long put position increase more than the proceeds from the short put position. As a result, the cost of the hedge should increase.*

*Comments:*
*Most students correctly observed that the cost increases, but only a small number explained that vega is positive, and that the vega of a long put is higher than that of a short put—hence the hedge cost rises.*

## 2. Learning Objectives:

2. The candidate will understand the principles and techniques for the valuation of derivatives.

**Learning Outcomes:**

(2a) Understand the principles of no-arbitrage and replication in asset pricing.

(2b) Understand Arrow-Debreau security and the distinction between complete and incomplete markets

(2d) Understand Stochastic Calculus theory and technique used in pricing derivatives including:

- Stochastic differential equations
- Ito integral
- Ito's Lemma
- Martingales,
- Change of numeraire
- Girsanov's theorem

(2e) Understand and apply the concepts of risk-neutral measure, forward measure, normalization, and the market price of risk

**Sources:**

Hull – Options, Futures and Other Derivatives (2021)
Campolieti – Financial Mathematics – A comprehensive treatment, Ch5.5, 2nd
Chin – Problems and Solutions in Mathematical Finance

**Commentary on Question:**

*The question was meant to test understanding of discounted price processes as examples of martingales and the application of the Radon-Nikodym derivative in option valuation.*

**Solution:**

(c)

(i) Calculate the value of $p$ such that the discounted price-process of $S$ is a martingale by evaluating the expectation of $\frac{1}{(1+r)^{t+1}} S_{t+1}$,

(ii) Verify your result using the definition of a martingale.

**Commentary on Question**:

*Candidates performed well overall. Typical mistakes included missing the discounting factor on either side of the martingale condition and attempting to show the stock price alone is a martingale, or not considering the other two conditions in the definition of a martingale*

**2. Continued**

For part (i), compute the conditional expectation of the next-period discounted price and impose the martingale condition:

$$E\left(\frac{S_{t+1}}{(1+r)^{t+1}}\middle|\mathcal{F}_t\right) = \frac{1}{(1+r)^{t+1}}\left(p\mu S_t + (1-p)\frac{S_t}{\mu}\right) = \frac{S_t}{(1+r)^t}.$$

Simplify to get:

$$\frac{1}{1+r}\left(p\mu + (1-p)\frac{1}{\mu}\right) = 1.$$

Now solve for $p$:

$$p = \frac{1+r-\frac{1}{\mu}}{\mu - \frac{1}{\mu}} = \frac{\mu(1+r)-1}{\mu^2-1}.$$

Alternatively, argue that the martingale condition implies the standard single-period risk-neutral formula:

$$p^* = \frac{1+r-d}{u-d}$$

with $u = \mu$ and $d = \frac{1}{\mu}$.

For part (ii), perform a consistency check by plugging the expression for $p$ into the martingale condition and verifying it holds, or mention it is true by construction of $p$.

Moreover, explain that:

- The discounted price process is adapted to the filtration: As a deterministic function of $S_t$, it is $\mathcal{F}_t$-measurable for all $t$ and regardless of $p$.
- The discounted price process is integrable, since $E(|S_t|) \leq S_0\mu^t < \infty$ for finite $t$ regardless of $p$.

(d) Derive the condition for there to be no arbitrage opportunity using the result from part (a).

**Commentary on Question**:
*Candidates performed adequately in this part. A common omission was to derive an inequality with no reference to arbitrage.*

**2. Continued**

For $p$ to be a valid (risk-neutral) probability, we need $0 < p < 1$. From (a)(i):

$$p = \frac{\mu(1+r)-1}{\mu^2-1} > 0 \implies \mu(1+r) > 1 \implies 1+r > \frac{1}{\mu}$$

and

$$p = \frac{\mu(1+r)-1}{\mu^2-1} < 1 \implies \mu(1+r)-1 < \mu^2-1 \implies 1+r < \mu.$$

Therefore, $\frac{1}{\mu} < 1+r < \mu$ and the risk-free return must lie strictly between the down factor $\frac{1}{\mu}$ and the up factor $\mu$. If $1+r \geq \mu$, shorting the stock and investing the proceeds at the risk-free rate produces a riskless arbitrage; if $1+r \leq \frac{1}{\mu}$, borrowing at the risk-free rate to buy the stock does the same. The inequalities need to be strict: $p = 0,1$ represents a degenerate measure under which one branch of the tree is impossible.

(c)

(i) State whether $\frac{1}{(1+r)^{t+1}} S_{t+1}$ is a martingale with respect to $\tilde{p}$.

(ii) Derive the value for the time *t+1* Radon-Nikodym derivative process, as measured at time t, with respect to the probability measures with corresponding "up" probabilities of *p* and $\tilde{p}$ .

**Commentary on Question**:
*Candidates performed poorly on this part. Two common errors were to not consider the case $r = 2/\mu$ and to only give the "up" value of the Radon-Nikodym derivative.*

For part (i), we consider the one-step expected return under $\tilde{p}$:

$$\tilde{p}\mu + (1-\tilde{p})\frac{1}{\mu} = \frac{\mu}{\mu-1} + \frac{\mu-2}{\mu(\mu-1)} = \frac{(\mu+2)(\mu-1)}{\mu(\mu-1)} = 1 + \frac{2}{\mu}.$$

The discounted price process is a martingale under $\tilde{p}$ if the above quantity equals $1+r$, hence $r = \frac{2}{\mu}$. In all other cases, it is not a martingale.

## 2. Continued

For part (ii), observe that the Radon-Nikodym derivative process over the step $t \to t+1$ is a random variable that takes one of two values, depending on whether the next move in the binomial tree is up or down:

$$\left.\frac{dP}{d\tilde{P}}\right|_{t\to t+1} = \begin{cases} \frac{p}{\tilde{p}} = p(\mu-1) & \text{"up"} \\ \frac{1-p}{1-\tilde{p}} = \frac{(1-p)(\mu-1)}{\mu-2} & \text{"down"} \end{cases}.$$

Giving the reciprocal Radon-Nikodym derivative is also fully correct, though not as useful for part (d) (ii).

(d)

(i) Calculate today's price of the option whose payoff in terms of $S_0$ is given as $\sqrt{S_2}$ if $\mu = 3, r = 0.05$.

(ii) Verify your result under the $\tilde{p}$ measure by direct calculation using part (c) (ii) above.

**Commentary on Question**:
*Candidates performed poorly on this part. Many candidates did not attempt (d) (ii) and others did not set up the calculation correctly.*

For part (i), calculate:

$$p = \frac{3(1.05)-1}{9-1} = 0.26875, \qquad 1-p = 0.73125.$$

Then compute:

$$E^p\left(\sqrt{S_2}\right) = \sqrt{S_0}\left(3p^2 + 2p(1-p) + \frac{(1-p)^2}{3}\right) = 0.78797\sqrt{S_0}.$$

Discounting back to time 0 gives:

$$V_0 = \frac{1}{(1.05)^2} E^p\left(\sqrt{S_2}\right) = 0.7147\sqrt{S_0}.$$

**2.** **Continued**

For (ii), use the Radon-Nikodym expression in (c) (ii) with $\tilde{p} = \frac{1}{\mu - 1} = 0.5$:

$$\frac{p}{\tilde{p}} = \frac{0.26875}{0.5} = 0.5375, \qquad \frac{1-p}{1-\tilde{p}} = \frac{0.73125}{0.5} = 1.4625.$$

By the Girsanov formula:

$$\begin{aligned} E^{p}\left(\sqrt{S_2}\right) &= E^{\tilde{p}}\left(\frac{dP}{d\tilde{P}}\sqrt{S_2}\right) \\ &= \sqrt{S_0}\left(3\tilde{p}^2\left(\frac{p}{\tilde{p}}\right)^2 + 2\tilde{p}(1-\tilde{p})\left(\frac{p}{\tilde{p}}\right)\left(\frac{1-p}{1-\tilde{p}}\right) \right. \\ &\quad \left. + \frac{(1-\tilde{p})^2}{3}\left(\frac{1-p}{1-\tilde{p}}\right)^2\right) = 0.78797\sqrt{S_0}. \end{aligned}$$

Then discount for two years, like in part (d) (i), to arrive at the correct answer.

**3.** **Learning Objectives:**

2. The candidate will understand the principles and techniques for the valuation of derivatives.

**Learning Outcomes:**

(2d) Understand Stochastic Calculus theory and technique used in pricing derivatives I including:

- Stochastic differential equations
- Ito integral
- Ito's Lemma
- Martingales,
- Change of numeraire
- Girsanov's theorem

(2f) Understand option pricing techniques including:

- Calculating an expectation
- By solving an PDE

**Sources:**

John Hull - Options, Futures, and Other Derivatives, Global Edition-Pearson (2021)
Problems and Solutions, Chin

**Commentary on Question:**

*This question tests candidates' knowledge on Ito's lemma and equity models. In general, candidates did well on this question.*

**Solution:**

(e)

(i) Verify that $B_1^2 = \int_0^1 2B_t\, dB_t + 1$.

(ii) Verify that $Cov\left(B_1^2, \int_0^1 e^{-t} B_t\, dB_t\right) = 2 - 4e^{-1}$.

(iii) Derive $Var\left[\int_0^1 e^{-t} B_t^2 dt\right]$.

**Commentary on Question**:

*Many candidates did well on this part by applying Ito's lemma. However, a few candidates applied deterministic calculus to prove this.*

(i) From Ito's lemma:

$$d(B_t^2) = 2B_t\, dB_t + dt \implies B_1^2 = \int_0^1 2B_t\, dB_t + 1.$$

## 3. Continued

(ii)

We have

$$Cov\left(B_1^2, \int_0^1 e^{-t} B_t \, dB_t\right) = E\left(B_1^2 \int_0^1 e^{-t} B_t \, dB_t\right)$$

since

$$E\left(\int_0^1 e^{-t} B_t \, dB_t\right) = 0.$$

Using part (a) (i):

$$E\left(B_1^2 \int_0^1 e^{-t} B_t \, dB_t\right) = E\left(\int_0^1 2B_t \, dB_t \int_0^1 e^{-t} B_t \, dB_t\right) + 0.$$

Now use Ito isometry to get:

$$E\left(\int_0^1 2B_t^2 \, e^{-t} dt\right) = \int_0^1 2E(B_t^2) \, e^{-t} dt = \int_0^1 2t \, e^{-t} dt = 2 - 4e^{-1}.$$

(iii)

Apply Ito's lemma to $e^{-t}B_t^2$ gives

$$d(e^{-t}B_t^2) = (-e^{-t}B_t^2 + e^{-t})dt + 2e^{-t}B_t dB_t$$

which implies

$$e^{-1}B_1^2 = \int_0^1 (-e^{-t}B_t^2 + e^{-t})dt + 2\int_0^1 e^{-t}B_t dB_t.$$

Rearranging terms gives

$$\int_0^1 e^{-t}B_t^2 dt = \int_0^1 e^{-t}dt - e^{-1}B_1^2 + 2\int_0^1 e^{-t}B_t dB_t = 1 - e^{-1} - e^{-1}B_1^2 + 2\int_0^1 e^{-t}B_t dB_t.$$

Taking variance of both sides, we get

$$Var\left[\int_0^1 e^{-t}B_t^2 dt\right] = e^{-2}\, Var[B_1^2] + 4\, Var\left[\int_0^1 e^{-t}B_t dB_t\right] - 4e^{-1}Cov\left(B_1^2, \int_0^1 e^{-t}B_t \, dB_t\right).$$

- $Var[B_1^2] = 2$ because $B_1 \sim N(0,1)$ therefore $B_1^2 \sim \chi^2(1)$ and its variance is two times the degrees of freedom. Alternatively, use the fact that $E[B_1^4] = 3$.
- Use Ito isometry:

$$4\, Var\left[\int_0^1 e^{-t}B_t dB_t\right] = 4\, E\left[\left(\int_0^1 e^{-t}B_t dB_t\right)^2\right] = 4\, E\left[\int_0^1 e^{-2t}B_t^2 dt\right] = 4\int_0^1 e^{-2t}\, \mathrm{E}(B_t^2)\, dt$$

$$= 4\int_0^1 t\, e^{-2t}\, dt = 1 - 3e^{-2}.$$

## 3. **Continued**

Putting everything together gives:

$$Var\left[\int_0^1 e^{-t} B_t^2 dt\right] = 2e^{-2} + 1 - 3e^{-2} - 4e^{-1}(2 - 4e^{-1}) = 1 - 8e^{-1} + 15e^{-2}.$$

(f) Critique the following stochastic model for the equity index:

$$dS_t = \mu\, dt + \sigma\, dB_t.$$

**Commentary on Question**:
*Most candidates did well on this part by noting that limitations of the model.*

The model is a generalized Wiener process (also called arithmetic Brownian motion). It is an inappropriate model for an equity index for the following reasons:

- The expected percentage return required by investors from a stock should be constant and independent of the stock's price. In the given model, the expected return drops as the stock price increases.
- the variability of the return in a short period of time should stay constant regardless of the stock price. This implies that the standard deviation of the stock price change in a short period of time should be proportional to the stock price itself. This is not true for the given model.

(c) Your manager makes the following statement:

"If the equity index follows the above model, then its expected continuously compounded return per annum realized over a period $[0, T]$ must be less than $\mu$."

Evaluate your manager's statement.

**Commentary on Question**:
*Most candidates correctly evaluated the statement.*

The statement is correct.

The expected continuously compounded return of the index over the period $[0, T]$ is

$$R = E\left[\frac{1}{T}\ln\frac{S_T}{S_0}\right] = \mu - \frac{\sigma^2}{2}$$

since

$$\ln\frac{S_T}{S_0} \sim \phi\left[\left(\mu - \frac{\sigma^2}{2}\right)T, \sigma^2 T\right].$$

This shows that the expected return is less than $\mu$.

## 3. Continued

(d) Calculate the 95% confidence interval of the index price in 6 months.

**Commentary on Question**:
*Many candidates obtained the correct confidence interval of the index price. A few candidates could not obtain the correct formula for the index price.*

The 95% confidence interval of the log index price is

$$\left[\ln S_0 + \left(\mu - \frac{1}{2}\sigma^2\right)T - 1.96\sigma\sqrt{T}, \quad \ln S_0 + \left(\mu - \frac{1}{2}\sigma^2\right)T + 1.96\sigma\sqrt{T}\ \right]$$

In 6 months' time, the 95% confidence interval of the index price is

$$\left[5750 \times \exp\left(0.5\left(0.1 - \frac{0.2^2}{2}\right) - 1.96 \times 0.2 \times \sqrt{0.5}\right), 5750 \times \exp\left(0.5\left(0.1 - \frac{0.2^2}{2}\right) + 1.96 \times 0.2 \times \sqrt{0.5}\right)\ \right]$$

which is $[4535.86,\ 7896.23]$

**4.** **Learning Objectives:**

2. The candidate will understand the principles and techniques for the valuation of derivatives.

**Learning Outcomes:**

(2g) Understand the limitations of the Black-Scholes-Merton model

(2h) Understand and apply numerical discretization methods to price options including Euler-Maruyama discretization and transition density methods

**Sources:**

Study note on interest rate calibration.

**Commentary on Question:**

*Candidates showed a fair understanding of the material tested in Question 4. Most were able to complete the main calculations with partial or full success. Strong candidates gave clear and well-organized solutions. Common issues included calculation mistakes and incomplete explanations. Overall performance was satisfactory, although only a smaller group demonstrated a strong overall understanding.*

**Solution:**

(a) Derive an expression for the probability $P[r_{t+s} < 0|r_t]$, where $r_{t+s}$ and $r_t$ are the interest rates at times $t+s$ and $t$ in years, respectively, with $s > 0$ and $t \geq 0$.

$r_{t+s}|r_t$ is a normally distributed random varibale with mean

$$E[r_{t+s}|r_t] = \bar{r} + (r_t - \bar{r})\, exp(-\gamma s)$$

And varinace

$$Var[r_{t+s}|r_t] = \frac{\sigma^2}{2\gamma}(1 - exp(-2\gamma s))$$

$$Z = \frac{r_{t+s} - E[r_{t+s}|r_t]}{\sqrt{Var[r_{t+s}|r_t]}}$$

is a standard normal randoma variable.

Therefore

$$P[r_{t+s} < 0|r_t] = \Phi\left(-\frac{E[r_{t+s}|r_t]}{\sqrt{Var[r_{t+s}|r_t]}}\right)$$

$$P[r_{t+s} < 0|r_t] = \Phi\left(-\frac{\bar{r} + (r_t - \bar{r})\, exp(-\gamma\ )}{\sqrt{\frac{\sigma^2}{2\gamma}(1 - exp(-2\gamma s))}}\right)$$

(b) Create a chart by plotting the values of $P[r_{t+s} < 0|r_t = 0.03]$ for $s = 0.1, 0.2, \ldots, 5$.

See excel worksheet

**4.** **Continued**

(c) Approximate the probability $P[r_{t+\bar{s}} < 0|r_t = 0.03]$ using the simulation.

See excel worksheet

(d) Compare the two methods in Parts (b) and (c) used to calculate the probability $P[r_{t+\bar{s}} < 0|r_t = 0.03]$

- **Part (b):** Theoretical maximum probability $P[r_{t+\bar{s}} < 0|r_t]$ was calculated.
- **Part (c):** An estimate of this probability was obtained using 100 simulated values of $r_{t+\bar{s}}$
- The transition density method was used, counting how many simulated values were negative.
- The estimated probability was computed as the ratio of negative values to total simulations.
- If the number of simulations is increased significantly, the estimated probability would converge to the theoretical value.

**5.** **Learning Objectives:**

1. The candidate will understand key types of derivatives.

2. The candidate will understand the principles and techniques for the valuation of derivatives.

3. The candidate will understand various applications and risks of derivatives.

**Learning Outcomes:**

(1c) Be able to compare European, American, Bermudan, Asian options, and various exotic options

(2a) Understand the principles of no-arbitrage and replication in asset pricing.

(3a) Understand the Greeks of derivatives

(3b) Understand static and dynamic hedging

(3c) Understand delta hedging, and the interplay between hedging assumptions and hedging outcomes

**Sources:**

John Hull - *Options, Futures, and Other Derivatives*, Global Edition-Pearson (2021)

- Ch. 13: Binomial Trees (Sections 13.1-13.3)
- Ch. 19: The Greek Letters
- Ch. 26: Exotic Options (26.3, 8-11, 13, 16, 17)

**Commentary on Question:**

*Parts (a) and (b) are straightforward, e.g., the calculation of time step, up-and-down factors and risk-neutral probability, testing a candidate's basic understanding of the basic concepts. The construction of the binominal tree in full 4-step requires a little bit deeper comprehension and generalization. Part (c) is an open-ended question. There are many strategies that can be acceptable.*

**Solution:**

(a)

(i) Calculate the up and down factors (*u, d*).

Up-and-down factors (*u*, *d*)

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.2\sqrt{0.25}} \approx 1.1052$$

$$d = \frac{1}{u} \approx 0.9048$$

# 5. **Continued**

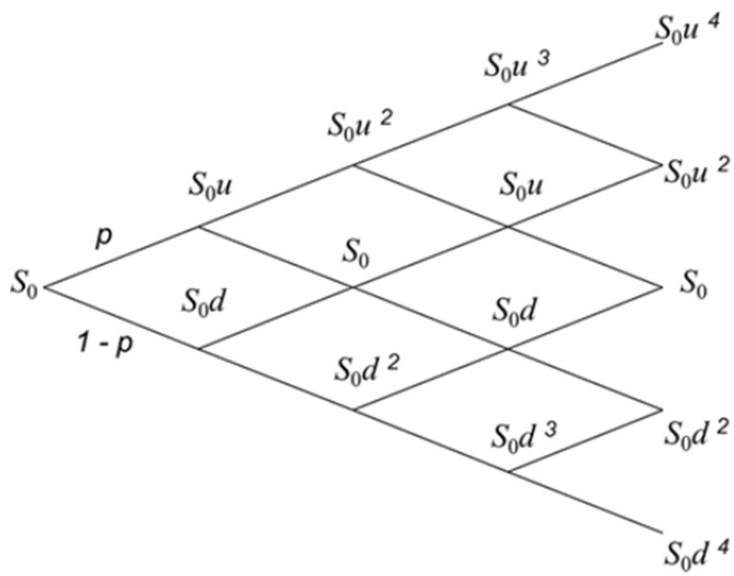
(ii) Calculate the risk-neutral probability ($p$).

Risk-neutral probability ($p$)

$$p = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.03*0.25} - 0.9048}{1.1052 - 0.9048} \approx 0.513$$

(iii) Sketch the binominal tree by showing the tree nodes with proper notations based on the calculations done in part (i) and part (ii).

Time step $\Delta t = 0.25$ (3 months)




(iv) Describe how to value the option using the binominal tree.

Based on the binominal tree constructed in (a)(i), we can value the Bermudan option through backward induction:

At each eligible exercise date (every 3 months), we compare the exercise value (K - S) with the continuation value (discounted expected future payoff).

1. At Maturity (T=1):
   - The option is exercised if $S_T < K$.
2. At Earlier Exercise Dates (T=0.25, 0.5, 0.75)
   - We check whether early exercise is optimal.
3. We step backward till we reach the beginning (T=0)

**5.** **Continued**

(v) Describe how to estimate the option delta and gamma.

**1. Delta (Δ):**

Delta is the sensitivity of the option price to changes in the stock price.

$$\Delta \approx \frac{P(S+\Delta S) - P(S-\Delta S)}{2\Delta S}$$

Make a small change (e.g., $\Delta S$=1) to stock spot price at T=0 and then calculate the Bermuda put option values, *P(101)* and *P(99)*, by updating all the tree nodes in the binominal tree. Delta can be approximated using the following formula:

$$\Delta \approx \frac{P(101) - P(99)}{2}$$

**2. Gamma (Γ):**

Gamma is the sensitivity of the option Delta to changes in the stock price.

$$\Gamma \approx \frac{\Delta(S+\Delta S) - \Delta(S-\Delta S)}{2\Delta S}$$

Apply the same method and the backward induction process to calculate the $\Delta(S+\Delta S)$ and $\Delta(S-\Delta S)$, as described above.

$$\Gamma \approx \frac{\Delta(S+\Delta S) - \Delta(S-\Delta S)}{2}$$

**Commentary on Question**:

*Apart from part (iv), candidates performed well in part (a). In part (iv), some candidates confused Measure P with Measure Q, some candidates forgot to consider discounting, and others overlooked early exercise. It is important for a candidate to point out this backward propagation process to arrive at the present value of the payoff while evaluating whether it is beneficial to exercise earlier along the way.*

(b) Calculate the total portfolio delta, gamma, and Vega, considering the quantities and positions (short/long).

Up-and-In Barrier Call (Short 100 contracts)

- **Delta:** 100 * (−0.06) = −6.0
- **Gamma:** 100 * (−0.01) = −1.0
- **Vega:** 100 * (−40) = −4000

Geometric Average Asian Call (Short 100 contracts)

- **Delta:** 100 * (−0.15) = −15.0
- **Gamma:** 100 * (−0.03) = −3.0
- **Vega:** 100 * (−9.5) = −950

**5. Continued**

Bermudan Put (Long 100 contracts)

- **Delta:** 100 * (−0.7) = −70.0
- **Gamma:** 100 * (0.025) = 2.5
- **Vega:** 100 * (25) = 2500

The total portfolio Greeks:

- **Delta: (−6.0) + (−15.0) + (−70.0) = -91.0**
- **Gamma: (−1.0) + (−3.0) + (2.5) = -1.5**
- **Vega: (−4000) + (−950) + (2500) = -2450**

**Commentary on Question**:
*Candidates performed very well in part (b). However, some candidates only considered the Up-and-In Barrier Call and the Geometric Average Asian Call, overlooking the Bermudan Put.*

(c)

(i) Describe a reasonable strategy to neutralize the portfolio delta, gamma, and Vega.

To neutralize the total portfolio delta, gamma, and Vega calculated in part (b), the strategy is to find a combination of European puts and calls, and stock shares to achieve the following Greeks:

- **Delta: +91.0**
- **Gamma: +1.5**
- **Vega: +2,450**

Due to the trading constraints – we can only trade whole integer option contracts and stock shares - we might not be able to perfectly neutralize the portfolio Greeks. Our recommended strategy is:

- **Prioritize Gamma and Vega neutrality.**
- **Because Gamma and Vega are second-order Greeks, we rely on Calls and Puts to minimize their residuals. If we cannot find a solution that perfectly neutralizes both simultaneously, we prioritize gamma neutrality over Vega neutrality because gamma is more critical for non-linear exposure and directly impacts delta.**
- **In the end, delta can be hedged with stocks and residual minimized.**

(ii) Calculate the positions to be traded to implement the strategy you described above. Justify your answer.

Given Greek Contributions per European Option (per contract):

| Option | Delta | Gamma | Vega |
|---|---|---|---|
| **Put (K=90)** | -0.22 | 0.015 | 30 |
| **Call (K=110)** | 0.41 | 0.02 | 40 |

- *x*= number of **Put (K=90)** contracts
- *y*= number of **Call (K=110)** contracts

Because the portfolio Gamma is negative (-1.50), we need to add positive Gamma using Europeans to achieve neutrality:

$$\begin{cases} 0.015x + 0.02y = 1.5 \ (to\ offset - 1.50) \\ 30x + 40y = 2450 \ (to\ offset - 2{,}450) \end{cases}$$

Apparently, there is no solution to the system above. This means we cannot find a solution to perfectly neutral gamma and Vega simultaneously, which is common in practice. Here are some options:

<u>Option 1</u>: If we choose y = 75 (buy 75 calls) and x=0 (0 puts), we can perfectly hedge gamma. The new portfolio Vega becomes +550 = (3000-2450) and the new portfolio delta becomes -60.25 = (-91 + 75 * 0.41). We need to buy 60 shares of stock to minimize delta to -0.25.

<u>Option 2</u>: If we choose x=100 (buy 100 puts) and y=0 (0 calls), we can perfectly hedge gamma. The new portfolio Vega becomes +550 = (3000-2450) and the new portfolio delta becomes -113 = (-91 + 100 * -0.22). We can buy 113 shares of stock to perfectly neutralize delta.

Because of the residual risk exposure, close monitoring and periodic rebalancing can be justified to prevent risk exposure from getting out of control. This is especially true for a portfolio loaded with exotic options.

**Commentary on Question**:
*Part (c) was not answered well and was the worst question in Q5. Many candidates overlooked the statement in the question, 'Assume that you cannot trade fractional option contracts, nor fractional stock shares,' and consequently failed to realize that a perfect hedge might not be feasible.*

**6.** **Learning Objectives:**

3. The candidate will understand various applications and risks of derivatives.

**Learning Outcomes:**

(3a) Understand the Greeks of derivatives

(3b) Understand static and dynamic hedging

(3c) Understand delta hedging, and the interplay between hedging assumptions and hedging outcomes

(3d) Understand the concepts of realized versus implied volatility

**Sources:**

- [John Hull - Options, Futures, and Other Derivatives, Global Edition-Pearson (2021)
- Ch. 19: The Greek Letters
- The Volatility Smile, Derman, Emanuel and Miller, Michael, 2016
- Ch. 3: Static and Dynamic Replication
- Ch. 5: The P&L of Hedged Option Strategies in a Black-Scholes-Merton World
- Ch. 6: The Effect of Discrete Hedging on P&L
- QFIQ-115-17: Which Free Lunch Would You Like Today, Sir?: Delta Hedging, Volatility Arbitrage and Optimal Portfolios

**Commentary on Question:**

*This question was intended to assess candidates' understanding of the relationship between actual and implied volatility, expected profit under different hedging assumptions, and the role of gamma in profit generation. Candidates were also expected to evaluate the impact of using alternative hedging volatilities, and to extend the analysis to the portfolio level by recognizing how expected profits aggregate across options and how diversification affects uncertainty.*

**Solution:**

(a) Explain why the arbitrageur expects to make a profit when hedging with implied volatility.

**Commentary on Question**:

*Most candidates performed well on this part. Candidates were generally able to identify that profits arise when actual volatility exceeds implied volatility. However, only a few candidates recognized that the profit is also attributable to gamma effects. Some candidates misinterpreted the question and discussed profits from hedging using actual volatility, which did not receive credit.*

## 6. Continued

Expected profit is $\frac{1}{2}(\sigma_a{}^2 - \sigma_i{}^2) \int e^{-r(T-t)} S^2 \Gamma^i \, dt$
or $d(\text{Profit}) = \frac{1}{2}(\sigma_a{}^2 - \sigma_i{}^2) e^{-r(T-t)} S^2 \Gamma^i dt$
When delta-hedging with implied volatility ($\sigma_i$), the profit comes from the difference between the actual realized volatility ($\sigma_a$) and the implied volatility used for hedging. The Black-Scholes assumes volatility is $\sigma_i$, but the stock moves with $\sigma_a$.

The profit is generated from the Gamma effect when $\sigma_a > \sigma_i$, the stock moves more than expected, and the delta-hedge is not perfectly adjusted.

However, the trader gains from rebalancing the delta hedge by buy-low and sell-high, due to higher realized volatility.

(b) Approximate the profit from this strategy.

**Commentary on Question**:
*Performance was generally weak on this part. Candidates did not recognize that gamma is time-dependent in this context. Some candidates performed calculations assuming gamma was independent of time; partial credit was awarded for correct application under this assumption. Some candidates incorrectly calculated expected profits based on hedging under actual volatility, which did not receive credit.*

$$\text{Expected Profits} = \frac{1}{2}(\sigma_a{}^2 - \sigma_i{}^2) \int_{t_0}^{T} e^{-r(t-t_0)} S^2 \Gamma^i \, dt$$

where $\Gamma^i = \frac{N'(d_1)}{S\sigma_i\sqrt{T-t}}$ ; $N'(d_1) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{d_1^2}{2}\right)$ ,
For ATM options, assuming $N'(d_1) = \frac{1}{\sqrt{2\pi}}$ and Gamma is simplified to
$\Gamma^i = \frac{1}{S\sigma_i\sqrt{2\pi(T-t)}}$ where $\Gamma^i$ is time-dependent.

$$\text{Expected Profits} = \frac{1}{2}(\sigma_a{}^2 - \sigma_i{}^2) \int_{t_0}^{T} e^{-r(t-t_0)} S^2 \frac{1}{S\sigma_i\sqrt{2\pi(T-t)}} \, dt$$

$$= \frac{1}{2}(\sigma_a{}^2 - \sigma_i{}^2) \int_{t_0}^{T} e^{-r(t-t_0)} S \frac{1}{\sigma_i\sqrt{2\pi(T-t)}} \, dt$$

To simplify the discount factor term, replace $e^{-r(t-t_0)}$ by $e^{-r(T-t_0)}$

$$= \frac{1}{2} \frac{(\sigma_a{}^2 - \sigma_i{}^2) S e^{-r(T-t_0)}}{\sigma_i\sqrt{2\pi}} \int_{t_0}^{T} \frac{1}{\sqrt{(T-t)}} \, dt$$

$$= \frac{S(\sigma_a{}^2 - \sigma_i{}^2)\, e^{-r(T-t_0)} \sqrt{T-t_0}}{\sigma_i\sqrt{2\pi}}$$

**6.** **Continued**

For ATM option, S = E, we derive the results of p.68 QFIQ-115-17.

$$\text{Expected Profit} = \frac{E(\sigma_a{}^2 - \sigma_i{}^2)\, e^{-r(T-t_0)}\sqrt{T-t_0}}{\sigma_i\sqrt{2\pi}}$$

Given that S = E = 100 (At the money call option)
T = 1, r =5%, $\sigma_i$ = 20%, $\sigma_a$ = 30%
$\text{Expected Profit} = \frac{100(0.3^2-0.2^2)\, e^{-0.05*1}\sqrt{1}}{0.2\sqrt{2\pi}}$ = 9.49

(c)

(i) Compare hedging with $\sigma_h$, $\sigma_a$, and $\sigma_i$.

(ii) Calculate the minimum and maximum possible profit if the arbitrageur hedges with $\sigma_h = 25\%$.

**Commentary on Question**:

*Candidates generally performed well on part (c)(i).*
*For part (c)(ii), performance was generally weak. Candidates who assumed that gamma is independent of time and proceeded to calculate the maximum term were awarded partial credit.*

c(i)
Hedging with $\sigma_a$
- The final profit is guaranteed and the P&L is known as Va – Vi, but how that is achieved is random.
- There is a draft term dX in the mark-to-market P&L. From a risk management perspective, this is not ideal.
- Trader is required to accurately forecast the actual volatility.

Hedging with $\sigma_i$
- Profit increment is deterministic as there are no draft term dX since $d(\text{Profit}) = \frac{1}{2}(\sigma_a{}^2 - \sigma_i{}^2)e^{-r(T-t)}S^2\Gamma^i dt$..
- No P&L fluctuation. This is a better behaved from risk management perspective.
- Trader do not need to know what actual volatility is exactly and just need to know greater or less than implied volatility.
- Profit is path-dependent, uncertain outcome.

Hedging with $\sigma_h$
- Balance smoothness and profit.
- It is a suboptimal hedging and path-dependent.

**6.** **Continued**

c(ii)
Similar to part b, the final profit is bounded by

$$V(S; t; \sigma_h) - V(S; t; \sigma_i) + \frac{E(\sigma^2 - \sigma_h{}^2)\, e^{-r(T-t_0)}\sqrt{T - t_0}}{\sigma_h\sqrt{2\pi}}$$

To compute the minimum profit,
Use Black-Scholes ATM calls,
V(100; 1; 0.25) – V(100; 1; 0.2)
$\approx 100 * (0.25 - 0.2) * \sqrt{1/2\pi}$ = 1.99

*Alternatively, candidate can use the full BS model to calculate*
*V(100; 1; 0.25) – V(100; 1; 0.2) = 12.34 – 10.45 = 1.89*

Calculate the maximum term:

$$\frac{E(\sigma^2 - \sigma_h{}^2)\, e^{-r(T-t_0)}\sqrt{T - t_0}}{\sigma_h\sqrt{2\pi}}$$

$$= \frac{100\left(0.3^2 - 0.25^2\right) e^{-0.05(1)}\sqrt{1}}{0.25\sqrt{2\pi}} = 4.17$$

Therefore, the minimum profit is 1.99 (or 1.89) and
the maximum profit is 1.99 + 4.17 = 6.17 (or 1.89 + 4.17 = 6.06)

(d) Describe how to find the expected profit at the portfolio level when hedging with implied volatilities.

**Commentary on Question**:
*Performance was average on this part. Most candidates stated that portfolio profits are additive but did not clearly describe the expected profit formula or how individual option contributions aggregate at the portfolio level.*

The profit from a portfolio is

$$\frac{1}{2}\sum_{k} q_k\left(\sigma^2 - \tilde{\sigma}_k{}^2\right)\int_{t_0}^{T_k} e^{-r(t-t_0)} S^2 \Gamma_k^i \, dt$$

Since only an option's gamma matters when we are hedging using implied volatility, calls and puts are effectively being treated the same from the expected profit perspective and the expected profit is additive.

## 6. Continued

Each option contributes based on its own gamma and maturity.
The expected profit is simply the sum of individual profits for each option, which is ½ of the present value of the Stock price squared ($S^2$) times gamma ($\Gamma^i$) at maturity times the difference between the actual volatility squared and the implied volatility squared times maturity.

The actual volatility $\sigma_a$ is the same, because all the options are on the same underlying stock. Note that because there is more than one expiration, the term $\Gamma^i$ can be zero for times beyond the expiration of the option.

(e) Assess whether diversification across strikes reduces the uncertainty of the expected profit.

**Commentary on Question**:
*Performance was average on this part. Some candidates were able to recognize the diversification effect across strikes, but explanations were often incomplete.*

Candidates might answer from either the gamma concentration perspective or the hedging error perspective.

<u>Gamma concentration and path-dependence</u>
Diversification reduces the uncertainty of the expected profit.
Different strikes have different gamma concentrations (diversification effect):
- ATM options have high gamma → more profit from volatility.
- OTM options have lower gamma → less profit but also less sensitivity to path.
A mix of strikes smooths the path-dependence effect, reducing the variance in the profit at the portfolio level.

<u>Hedging error</u>
The arbitrager delta-hedges discretely at implied volatility, not only will his P&L be path-dependent, but also his P&L will pick up a random component that occurs because the hedge is accurate only instantaneously, but not during intervals between rebalancing.
Having a diversified portfolio of options with different strikes smooths out the path-dependence and randomness.
Thus, it helps mitigate the risk of the expected profit at the portfolio level.

**7.** **Learning Objectives:**

3. The candidate will understand various applications and risks of derivatives.

**Learning Outcomes:**

(3f) Identify and evaluate embedded options in liabilities (e.g., indexed annuity, structured product based variable annuity, and variable annuity guarantee riders including GMxB, etc.)

(3g) Demonstrate an understanding of hedging for embedded option in liabilities with:

- Risks that can be hedged, including those of equity, interest rate, volatility, and cross Greeks,
- Risks that can only be partially hedged or cannot be hedged, including those of policyholder behavior, mortality, basis, counterparty, correlation, and operational failures

**Sources:**

INV201-105-25: An Introduction to Computational Risk Management of Equity-Linked Insurance, Feng, 2018 (sections 1.2-1.3, 4.7 & 6.2-6.3)

INV201-107-25: It's RILA time: An introduction to registered index-linked annuities (excluding Appendices)

**Commentary on Question:**

*Commentary listed underneath question component.*

**Solution:**

(a)

(i) Explain the differences between TVA and RILA in in terms of Basis risk.

(ii) Explain why hedging is more effective for RILA than for TVA.

**Commentary on Question**:

*Candidates performed below expectation on this question. For part (i), many listed general differences between TVA and RILA rather than focusing on differences with Basis risk implications.*

(i) TVAs:

- Direct investment of premiums into available mutual funds selected by the policyholder
- Long-term embedded guarantees
- Require hedging instruments such as futures and exchange-traded funds (since the underlying mutual funds cannot be shorted)

# 7. Continued

RILAs:

- o Premiums are not directly invested in the selected indices
- o Short-term guarantees
- o Liabilities can be decomposed into European options on major equity indices

(i) Hedging TVA liabilities is relatively complex, as basis risk limits the effectiveness of available market instruments to hedge the embedded guarantees. The long-term liabilities are also highly dependent on policyholder behavior. RILA guarantees, on the other hand, are shorter-term and linked to the performance of major equity indices. This allows insurers to hedge these exposures with near-perfect precision.

(b) Write down the no-arbitrage time-0 price of the GMAB gross benefit payment, $PV(G_{T_1}, t = 0)$, in the form of Black-Scholes formula using the variables defined in the assumptions above and the d1, d2 functions below:

$$d_1(T, x) = \frac{\ln(x) + (r - m + \sigma^2/2)T}{\sigma\sqrt{T}}$$
$$d_2(T, x) = d_1(T, x) - \sigma\sqrt{T}$$

**Commentary on Question**:
*Most candidates identified the appropriate formula to apply to this question, but simply wrote it as a final answer. However, the question specified to provide a price for $PV(G_{T_1}, t = 0)$, which is the gross benefit payment, rather than the gross liability for the insurer.*

The GMAB gross benefit, as defined in the problem, can be decomposed into the payoff from holding the fund and a put on the fund.

$$\begin{aligned} PV(G_{T_1}, t = 0) &= {}_{T_1}P_x\, \mathbb{E}\left[e^{-rT_1}\max(G_0,\ F_{T_1})\right] \\ &= {}_{T_1}P_x\, e^{-rT_1}\mathbb{E}\left[\, F_{T_1} + \max(0,\ G_0 - F_{T_1})\right] \\ &= {}_{T_1}P_x e^{-rT_1}\left(\mathbb{E}\left[\, F_{T_1}\right] + \mathbb{E}\left[\max(0,\ G_0 - F_{T_1})\right]\right) \\ &= {}_{T_1}P_x\, e^{-rT_1}\left( F_0 e^{(r-m)T_1} + G_0 \Phi\left(-d_2(T_1, \frac{F_0}{G_0})\right) \right. \\ &\quad \left. - F_0 e^{(r-m)T_1}\Phi\left(-d_1(T_1, \frac{F_0}{G_0})\right)\right) \end{aligned}$$

**7. Continued**

The final line follows from formula (4.53) from the Formula Sheet.

Alternate solution:
The gross benefit as defined in the problem can be decomposed into the fixed initial guarantee, $G_0$, and a call on the fund value.

$$\begin{aligned} PV(G_{T_1}, t=0) &= {}_{T_1}P_x\, \mathbb{E}\left[e^{-rT_1} \max\left(G_0,\ F_{T_1}\right)\right] \\ &= {}_{T_1}P_x\, e^{-rT_1}\mathbb{E}\left[G_0 + \max\left(0,\ F_{T_1} - G_0\right)\right] \\ &= {}_{T_1}P_x e^{-rT_1}\left(G_0 + \mathbb{E}\left[\max\left(0,\ F_{T_1} - G_0\right)\right]\right) \\ &= {}_{T_1}P_x\, e^{-rT_1}\left(G_0 + F_0 e^{(r-m)T_1}\Phi\left(d_1(T_1, \frac{F_0}{G_0})\right)\right. \\ &\left. - G_0\Phi\left(d_2(T_1, \frac{F_0}{G_0})\right)\right) \end{aligned}$$

(c)

(i) Write down the time-0 value of GMAB benefit policy that is currently in-force and will enter a GMDB policy upon maturity (denoted as $B_{GMAB,the\ \ GMDB}(t, F_t)$).

(ii) Derive the formula of the GMDB rider charge, $m_d$, under the no-arbitrage assumption.

**Commentary on Question**:
*Candidates performed poorly on this question. Most skipped entirely or failed to appropriately integrate the payoff provided in the Hint. For (ii), some candidates were able to provide a form for the PV of the GMDB rider charges or highlight the no-arbitrage principle to apply without explicitly solving (i).*

(i) The time-0 value of $B_{GMAB,then\ GMDB}(t, F_t)$) will be the discounted expectation of the payoff provided in the Hint. In addition, to be able to convert from GMAB to GMDB, policyholder needs to survive to $T_1$, so we assume ${}_{T_1}P_x = 1$.

## 7. **Continued**

$$B_{GMAB,then\ GMDB}(0, F_0) = \mathbb{E}\left\{\int_{T_1}^{T_2} e^{-rs}\left[\max(G_0, F_{T_1})(1-\frac{F_s}{G_{T_1}})_+\right] {}_sP_x\,\mu_{x+s}ds\right\}$$

$$= \left\{\int_{T_1}^{T_2} e^{-rs}\mathbb{E}\left[\max\left(G_0,\ F_{T_1}\right)|F = F_0\right]\right.$$

$$\left. *\ \mathbb{E}\left[(1-\frac{F_s}{G_{T_1}})_+\right] {}_sP_x\,\mu_{x+s}ds\right\}$$

The two expectations can be separated given non-overlapping periods.

The first expectation was derived in part (b):

$$\mathbb{E}\left[\max\left(G_0,\ F_{T_1}\right)|F = F_0\right] = F_0e^{(r-m)T_1} + G_0\Phi\left(-d_2\left(T_1,\frac{F_0}{G_0}\right)\right) - F_0e^{(r-m)T_1}\Phi\left(-d_1\left(T_1,\frac{F_0}{G_0}\right)\right)\ (*)$$

The second expectation has the same payoff as a put option, with strike 1, on $\frac{F_s}{G_{T_1}}$.

$$\mathbb{E}\left[\left(1-\frac{F_s}{G_{T_1}}\right)_+\right] = \Phi\left(\frac{m+\frac{\sigma^2}{2}-r}{\sigma}\sqrt{s-T_1}\right) - e^{(r-m)(s-T_1)}\Phi\left(\frac{m-\frac{\sigma^2}{2}-r}{\sigma}\sqrt{s-T_1}\right)(**)$$

$$B_{GMAB,then\ GMDB}(0, F_0) = \int_{T_1}^{T_2} e^{-rs}\left[F_0e^{(r-m)T_1} + G_0\Phi\left(-d_2\left(T_1,\frac{F_0}{G_0}\right)\right) - F_0e^{(r-m)T_1}\Phi\left(-d_1\left(T_1,\frac{F_0}{G_0}\right)\right)\right] * \left[\Phi\left(\frac{m+\frac{\sigma^2}{2}-r}{\sigma}\sqrt{s-T_1}\right) - e^{(r-m)(s-T_1)}\Phi\left(\frac{m-\frac{\sigma^2}{2}-r}{\sigma}\sqrt{s-T_1}\right)\right] {}_sP_x\,\mu_{x+s}ds$$

## 7. Continued

(ii) From the insurer's perspective, the fee should be set such there is no arbitrage, i.e., $B_{GMAB,then\ GMDB}(0, F_0) = P_d(0, F_0)$, where $P_d(0, F_0)$ is the PV of the GMDB rider fee.

$$P_d(0, F_0) = \mathbb{E}\left\{\int_{T_1}^{T_2} e^{-rs}\, m_d\, {}_sP_x F_s ds\right\} = m_d\, \mathbb{E}\left\{\int_{T_1}^{T_2} e^{-rs}\, {}_sP_x F_s ds\right\}$$

Therefore, $B_{GMAB,then\ GMDB}(0, F_0) = m_d\, \mathbb{E}\left\{\int_{T_1}^{T_2} e^{-rs}\, {}_sP_x F_s ds\right\}$

$$m_d = \frac{B_{GMAB,then\ GMDB}(0, F_0)}{\mathbb{E}\left\{\int_{T_1}^{T_2} e^{-rs}\, {}_sP_x F_s ds\right\}}$$

(d) Calculate the fair cap rate at which the present value of the RILA product equals the initial premium.

**Commentary on Question**:
*Candidates performed well on this part. Most common errors included using the wrong sign for combining option prices to find total option cost, or using goal seek to solve for a zero-option cost as opposed to zero-profit.*

**RILA**

| | |
|---|---|
| Underlying Asset – Current Price ($S_0$) | 100 |
| Dividend Yield (q) | 0% |
| Implied Volatility (σ) | 15% |
| Term (t) | 3 |
| Risk-Free Rate (r) | 4% |
| Buffer Rate% (B) | 10% |
| Cap Rate% (C) | **44.52%** <- answer |
| Participation rate ( $P^{IR}$ ) | 100% |

| | Type | Positions | Option Value |
|---|---|---|---|
| Option 1 | ATM Call | Long | 16.420 |
| Option 2 | OTM Put | Short | 2.461 |
| Option 3 | OTM Call | Short | 2.651 |

| | |
|---|---|
| Sum of option cost | 11.308 |
| Option Budget | 11.308 |
| Profit | $0.00 |

(e) The company anticipates an economic downturn in the future that will cause the index volatility to move from 15% to 18%, which will reduce the option cost (value of the above three European options).

Explain how it could impact the calculated cap rate.

**Commentary on Question**:
*Candidate performance was neutral on this part. Some candidates seemed unsure of the meaning of "option cost," i.e., referring to higher/lower cost of each option as opposed to the value of the combined position. For full credit, candidates needed to explain their conclusion with complete logic. Limited credit given to candidates who used the Excel workbook to reach their conclusion.*

**7.** **Continued**

All options increase in value due to the higher volatility. However, the OTM options increase in value more than the ATM Call, lowering the overall option cost. Therefore, the cap rate can be increased for the OTM Call to maintain the same option budget.