

CP 351 Model Solutions

March 2026

1. Learning Objectives:

1. The candidate will understand the objectives of Asset Liability Management (ALM).

Learning Outcomes:

- (1b) Understand the lessons learned from ALM failures.
- (1c) Demonstrate an understanding of various ALM risks.
- (1d) Describe how different pension and insurance contracts generate embedded options.

Sources:

CP351-106-25: LDI Explained - BMO Global Asset Mgmt, Ch. 1-2

What Can Insurers and Pension Funds Learn from Bank Failures

Commentary on Question:

Candidates did well on this first question with average score of 3.5 out of 6, or 58% above average score. It is a fair and relatively easy question to answer; it also helps candidates ease exam anxiety at the beginning and gain confidence to continue working on the rest of questions.

Question #1 is intended to examine candidates' understanding of ALM risks and what instruments/tools can be used to manage the risk. It has a good mix of Retrieval/Comprehension and Analysis/Knowledge Utilization in terms of cognitive level, while the latter parts (Part C and D) are relatively challenging than the former parts (Part A and B). They are still a straightforward application of the basic ALM definition and concepts.

Solution:

- (a) Identify two main unrewarded risks that DEF Pension Scheme faces.

Commentary on Question:

Candidates did well on the part with average score more than 70% above average, this is directly related to study material; most candidates were able to list the risks, but the details explanation following the risks varied by candidates, which can help distinguish candidates' understanding.

1. Continued

Candidates correctly identify the two risks and accurately describe the mechanics as how the risks impact the liabilities received full credits. Missing details or describing unrelated issues lose credit.

1. Interest rate risk: The present value of a scheme's liabilities depends on the discount rate used to value them. This value varies as interest rates/discount rates change. This means that the present value of the liabilities can change independently of changes in the scheme's asset value, giving rise to interest rate risk. All of the scheme's liabilities must be discounted back to present value and are therefore sensitive to changes in interest rates. It is a regulatory requirement to link the discount rate to a market-based rate of interest, such as long-term gilt or swap yields thus, as these yields change the value of a scheme's liabilities will also change.
 2. Inflation risk: Changes in inflation will affect the amount a scheme pays out when a liability falls due. When valuing inflation linked liabilities the actuary will apply an assumption about future anticipated inflation. If the level of anticipated inflation changes this will have a direct knock-on effect on the value of any inflation linked liabilities. Changes in inflation expectations can therefore have a knock-on effect on the value of a scheme's liabilities independent of any changes to a scheme's asset value, giving rise to inflation risk.
- (b) Explain how the three investment choices above can help DEF mitigate the unrewarded risks identified in Part (a).

Commentary on Question:

This is a straightforward question with average score more than 80% above average. Most candidates understand the listed instruments well and were able to articulate how to use them to mitigate the risks from the answer of Part A. In fact, from hindsight, Part B provides a hint to answer Part A if candidates knew the listed instruments, at least for inflation risk piece. Candidates correctly targeted each instrument to the risks in Part A and explained the connections received full credits. Missing details, providing contradictory statements or describing unrelated issues lose credit.

Fixed rate bonds can help mitigate interest rate risk for the portion of DEF's liabilities that are not linked to inflation (nominal liabilities). By holding fixed rate bonds with durations that closely match the duration of these liabilities, the change in bond values in response to interest rate movements can offset the corresponding change in the present value of nominal liabilities, thereby reducing surplus volatility.

1. Continued

Inflation-linked bonds are useful for mitigating inflation risk for the portion of DEF's liabilities that are inflation-linked. Since the principal and coupon payments of these bonds rise with inflation, their value will move in line with the inflation-adjusted liabilities. This helps ensure that increases in inflation expectations, which would otherwise increase the value of inflation-linked liabilities, are offset by increases in the value of the scheme's assets.

Interest rate swaps can also be used to manage interest rate risk without the need to fully fund the purchase of physical bonds. By entering into pay-fixed/receive-floating swaps, DEF can synthetically increase the interest rate sensitivity (duration) of its assets to better match that of its liabilities. Swaps allow for more flexibility and capital efficiency in implementing a liability-matching strategy, particularly when the scheme needs to adjust its hedge ratio without large cash outlays.

- (c) Calculate the PV01 of the zero-coupon bond.

Commentary on Question:

This question has a clear cut in terms of right or wrong. Candidates received 36% of average score on this part, which indicates that it is fair but a bit challenging than Part A and B. Candidates who applied the PV01 formula correctly received full credit, those who used the Duration as an approximation arrived at similar answer received partial credit.

Bond details: 20-year zero-coupon bond, face value £100 million, yield 3%.

Present value (PV):

$$PV = (\text{Face Value}) / (1 + \text{yield})^n = 100,000,000 / (1 + 0.03)^{20} \approx £55,367,575$$

New PV after a 0.01% yield increase (to 3.01%):

$$PV_{\text{new}} = (\text{Face Value}) / (1 + \text{new yield})^n = 100,000,000 / (1 + 0.0301)^{20} \approx £55,260,175$$

PV01:

$$PV01 = PV - PV_{\text{new}} = £55,367,575 - £55,260,175 = £107,400$$

The bond's PV01 is £107,400, meaning its value decreases by £107,400 for a 0.01% rate increase.

[Duration approximation for PV01: (partial credits)]

Present value (PV):

$$PV = (\text{Face Value}) / (1 + \text{yield})^n = 100,000,000 / (1 + 0.03)^{20} \approx £55,367,575$$

20-year zero-coupon bond **Macaulay** duration = 20, or

20-year zero-coupon bond **Modified** duration = $20 / (1 + 0.03) = 19.42$,

1. Continued

PV01:

$$\text{PV01} = \text{PV} * \text{Duration (e.g Modified)} * 0.0001 \text{ (1 basis point)} = \\ 55,367,575 * 19.42 * 0.0001 = 107,509$$

- (d) Calculate the notional amount of the swap to fully hedge the liability's PV01 along with the bond.

Commentary on Question:

This question also has a clear cut in terms of right or wrong. Candidates received about 37% of average score on this part, which indicates that it is fair but has similar challenges as Part C. Candidates who applied the PV01 result correctly and found the right notional amount of swap received full credit, those who used the Duration as an approximation in Part C, but arrived at similar answer and could get the right answer if the right answer in Part C is used also received full credit. Partial credits were given when certain interim answers were correctly calculated.

Liability details: PV01 = £160,000 = $80000000 * 20 / 10000$ per 0.01% rate increase.

Bond PV01: £107,400.

Remaining PV01 to hedge: $160,000 - 107,400 = \text{£}52,600$

Swap notional needed:

$$\text{Notional} = (\text{Remaining PV01}) / (\text{PV01 per £1 million}) \\ = 52,600 / 400 = \text{£}131.5 \text{ million}$$

2. Learning Objectives:

2. The candidate will understand how to measure risks from assets and liabilities.

Learning Outcomes:

- (2c) Evaluate the difficulties of investing for long-tail liabilities (i.e. beyond 30 years) such as inflation and longevity.

Sources:

CP351-101-25: ALM for Life, Annuities, and Pensions, Hatfield (2024), Section 6

CP351-110-25: New Frontiers: Backing Long-term Insurance Liabilities with Non-fixed-income Assets

Commentary on Question:

This question covers derivative-based duration management, segment-level hedging analysis, and evaluation of a non-fixed-income carve-out strategy under LICAT.

Candidates generally did well, with stronger results on parts (a), (c), and (d), and more difficulty on parts (b) and (e).

Full credit required candidates to explain how their recommended instruments work mechanically, not just name them. For part (b), the conditions needed to be tied to specific valuation or earnings consequences. For part (e), all three sub-topics had to be addressed with the Canadian regulatory context in mind.

Candidates did better on the quantitative parts (c) and (d) than the qualitative parts. Part (a) was generally well-answered, though many candidates did not explain how the forward-starting swap actually extends duration. In part (b), many candidates cited generic market risk without explaining how it would affect earnings or valuation specifically. Part (e) was the most difficult; most candidates handled the interest rate mismatch section reasonably well but struggled with the LICAT capital implications.

Solution:

- (a) Recommend a structured investment approach using derivatives that allows the investment team to maintain exposure to the preferred 10-year corporate bonds while meeting the ALM guidelines.

Commentary on Question:

Candidates did well on this part. Most identified an appropriate derivative instrument, though some did not explain how the forward-starting swap extends duration in practice. Candidates who explained both the instrument and why it works within the ALM framework received full credit.

2. Continued

A structured approach would be to purchase the 10-year corporate bonds and simultaneously enter into a 10-in-10 forward starting receiver swap. After 10 years (when the bond matures), the swap begins paying fixed and receiving floating for another 10 years, bringing the total duration to 20 years in line with the ALM guideline, while keeping the team's preferred credit exposure in the 10-year space.

Alternatively, if credit exposure needs to be managed separately, the team could buy long-duration Treasuries for duration matching and sell 5-year CDS protection on chosen credits to gain credit spread exposure, rolling the CDS every 5 years.

- (b) Identify three conditions under which this strategy might backfire from a valuation or earning perspective, even if the duration matching is successful.

Commentary on Question:

Candidates found this part more challenging. Many cited generic derivative risk without identifying conditions specific to valuation or earnings. Simply naming a risk was not enough; candidates needed to explain the financial reporting or valuation consequence to receive full credit.

1. Accounting mismatch: If hedge accounting treatment is not available, changes in the market value of the derivatives (e.g., swaps or CDS) may flow through earnings even if they are economically justified, causing earnings volatility even when the duration hedge is working as intended.

2. Mark-to-market losses during rate movements: If interest rates rise sharply, the value of the forward starting receiver swap would fall, creating mark-to-market losses. Even if higher reinvestment yields economically offset this, accounting may not capture that offset, resulting in a reported valuation loss.

3. Credit events on CDS: If selling CDS protection, a credit event (e.g., downgrade or default) would result in a loss payout even though no bond was directly held. This could result in a material loss if the notional CDS exposure is large. Other valid points include basis risk from non-parallel rate shifts and collateral calls under rising rate scenarios.

- (c) Calculate the leverage ratio for each segment.

Commentary on Question:

Most candidates applied the leverage ratio formula correctly. See the attached Excel for the model calculations.

See the attached Excel spreadsheet for full calculations. Segment L1: Leverage Ratio = $\$400\text{M} / (\$450\text{M} - \$400\text{M}) = 8$. Segment L2: Leverage Ratio = $\$300\text{M} / (\$200\text{M} - \$300\text{M}) = -3$ (negative equity; segment is underfunded).

2. Continued

- (d) Assess whether each segment is under-hedged or over-hedged.

Commentary on Question:

Candidates performed very well on this part. Most correctly assessed the hedging status of each segment. See the attached Excel for model calculations. Either actual duration or dollar duration approach is acceptable.

See the attached Excel spreadsheet for full calculations. Using actual duration: L1 underhedged (asset duration 8 < liability duration 10); L2 overhedged (asset duration 40 > liability duration 35). Using dollar duration: both L1 and L2 are underhedged. Either approach is acceptable.

- (e) Evaluate this proposal by addressing the followings:
- (i) Interest rate mismatch
 - (ii) Liquidity and product design
 - (iii) Capital implications under the Canadian Life Insurance Capital Adequacy Test (LICAT)

Commentary on Question:

Candidates generally handled the interest rate mismatch section reasonably well but struggled more with liquidity/product design and LICAT capital implications. Candidates tended to do better on the liquidity sub-question than the interest rate mismatch and LICAT sections. Full credit required covering all three areas with reference to the Canadian regulatory context.

i. Interest Rate Mismatch: Real estate and infrastructure assets are not interest rate instruments and do not provide direct interest rate sensitivity aligned with the liabilities they back. Under the LICAT framework, this mismatch increases the interest rate risk charge because these assets do not hedge long-duration liability exposures. Some NFI assets may generate interest-sensitive income, but the hedging effect is indirect and imperfect. This is especially problematic given the long-duration nature of the participating liabilities, where even modest rate movements can have a significant impact on liability values.

ii. Liquidity and Product Design: Real estate and infrastructure are illiquid assets that typically require long holding periods to realize their economic benefits. Participating policies often contain contractual or behavioral uncertainties such as policyholder surrenders, dividend options, or changing premium patterns. These features could generate liquidity needs that are not matched by the asset side if cash flows are unexpectedly front-loaded. Without careful structuring or dedicated liquidity buffers, the insurer could face asset-liability cash flow

mismatches, potentially requiring forced liquidation of illiquid assets at unfavorable terms.

iii. LICAT Capital Implications: From a capital perspective, carve-out assets result in increased required capital due to both interest rate risk (as discussed above) and market risk, based on the volatility and return profile of NFI asset classes. Compared to equities, infrastructure and real estate are somewhat favored under LICAT due to lower historical volatility and stable income generation, which helps reduce the market risk factor. However, these asset classes still carry higher capital charges than long-duration fixed income, so the insurer must weigh the potential return benefit against the higher capital requirement. This trade-off should be assessed carefully given OSFI's expectations and the impact on the insurer's LICAT ratio.

3. Learning Objectives:

2. The candidate will understand how to measure risks from assets and liabilities.

Learning Outcomes:

- (2b) Evaluate a company's or a portfolio's exposures to various risks.
- (2c) Evaluate the difficulties of investing for long-tail liabilities (i.e. beyond 30 years) such as inflation and longevity.

Sources:

CP351-101-25: ALM for Life, Annuities, and Pensions, Hatfield (2024), sections 4 & 6

CP351-110-25: New Frontiers: Backing Long-term Insurance Liabilities with Non-fixed-income Assets

Commentary on Question:

Candidates generally performed well in computing the effective duration and convexity but struggled with the other computational aspects of the problem.

Solution:

- (a) Calculate the following for the assets in the immunizing portion of the portfolio:
 - (i) Effective duration
 - (ii) Effective convexity
 - (iii) Required capital amount

Commentary on Question:

Most candidates correctly found the effective duration and convexity. Few candidates were able to correctly determine the required capital amount.

Please see Excel sheet.

- (b) Assess the portfolio manager's comment.

Commentary on Question:

Both responses supporting or rejecting the portfolio manager's comment were acceptable as long as there was support backing up the position taken in the response. This support had to reference expected return or risk and comment on the effect of inflation to receive full credit.

While the portfolio manager's recommendation focuses on the higher equity risk premium of equity A, a comprehensive analysis must account for the trade-off between excess return, heightened volatility, and sensitivity to the inflation assumptions.

3. Continued

1. Risk-Return Analysis

Equity A offers a higher equity risk premium, suggesting a higher potential for total return. However, this is accompanied by a higher standard deviation. When evaluated using the Sharpe Ratio, equity A appears more efficient than equity B, but the efficiency assumes the investor can tolerate the higher probability of drawdowns.

2. Sensitivity to Inflation assumptions.

The manager's recommendation is contingent on the assumption that inflation remains stable at 4%. This creates two primary risks:

- High volatility equities often exhibit greater sensitivity to macroeconomic shifts. If inflation deviates from the 4% target, equity A is likely to experience more aggressive price swings than equity B.
- If inflation rises unexpectedly, the real risk-free rate and discount rate may increase, potentially compressing the equity risk premium.

- (c) Calculate the total risk of the portfolio.

Commentary on Question:

Candidates struggled to obtain the correct response in part (c).

See Excel sheet.

- (d) Calculate the total leverage ratio that the carve-out portion of the portfolio utilizes.

Commentary on Question:

Many candidates did not correctly find the total leverage ratio.

See Excel sheet.

4. Learning Objectives:

2. The candidate will understand how to measure risks from assets and liabilities.

Learning Outcomes:

- (2a) Demonstrate an understanding of various risk identification tools.

Sources:

Fixed Income Securities: Tools for Today's Markets, Fourth Edition. Tuckman and Serrat, Ch. 4

CP351-101-25: ALM for Life, Annuities, and Pensions, Hatfield (2024), section 4

CP351-107-25: Key Rate Durations: Measures of Interest Rate Risk

Commentary on Question:

This question assessed proficiency in interest rate risk measures, specifically duration and convexity. While most candidates understood the broad concept of duration as the sensitivity of value to rate changes, many struggled to apply this across different forms, such as key rate measures or derivative-based definitions. High-scoring responses were distinguished by a deeper grasp of these various facets, demonstrating the technical rigor to transition accurately between general concepts, simplified formulas, and specific calculations.

Solution:

- (a) Calculate the effective duration of the liability.

Commentary on Question:

Generally well-answered, although a minority of candidates confused the concepts of key rate duration and total duration.

See Excel file.

- (b) Write down a formula for the following in terms of $V(x)$, a , b , and x :
 - (i) Effective duration
 - (ii) Effective convexity

Commentary on Question:

Candidates appeared less certain of the derivative-based definitions for duration and convexity. While many could identify the basic derivatives, few correctly applied the $-1/V$ and $1/V$ multipliers. Additionally, reliance on the central difference formula without proper simplification resulted in incomplete or overly complex responses.

4. Continued

$$\text{Duration} = -\frac{1}{V} \frac{\partial V}{\partial x} = \frac{-2a(x+2)}{V}$$
$$\text{Convexity} = \frac{1}{V} \frac{\partial^2 V}{\partial x^2} = \frac{2a}{V}$$

- (c) Calculate the convexity of the liability.

Commentary on Question:

Success in this part required the correct foundation from the previous section. A significant number of candidates were unable to achieve higher marks here due to fundamental errors in their part (b) derivations.

See Excel file.

- (d) Calculate the yield-based duration of Bond Y. (Use the “Goal Seek” function as needed.)

Commentary on Question:

While the majority of candidates demonstrated proficiency in the core calculations, two common errors were observed. Some candidates incorrectly provided the Macaulay duration as their final response, while others stated the yield itself rather than the yield-based duration measure requested by the prompt

See Excel file.

5. Learning Objectives:

3. The candidate will understand tools and strategies to manage ALM risks.

Learning Outcomes:

- (3b) Demonstrate an understanding of and apply tools and techniques for measuring and managing interest rate risk in an ALM context.
- (3c) Apply stress testing and scenario analysis to assess extreme ALM events.
- (3e) Explain and implement techniques used to mitigate market risks.

Sources:

CP351-106-25: Liability Driven Investment Explained, BMO Global Asset Management

CP351-101-25: ALM for Life, Annuities, and Pensions, Hatfield (2024), section 3

CP351-109-25: IAIS Application Paper on Liquidity Risk Management

CP351-110-25: New Frontiers: Backing Long-term Insurance Liabilities with Non-fixed-income Assets

Quantitative Enterprise Risk Management, Hardy, Mary and Saunders, David, 2022 Ch. 8 – Market Risk Models

Quantitative Enterprise Risk Management, Hardy, Mary and Saunders, David, 2022 Ch. 7 – Stress Testing

Commentary on Question:

Commentary listed underneath question component.

Solution:

- (a) Describe three challenges TJK faces in ALM as it relates to long-term liabilities.

Commentary on Question:

To receive full credit, candidates must cover all 3 major challenges (interest rate risk, scarcity of assets and accurate projection of cash flow). Candidates provide multiple risks that relate to the projection of cash flow will only receive partial credit.

- Interest Rate Risk: Changes in interest rates can lead to large impact on long-term cash flow.
- Scarcity of Assets: The longer the investment maturity, the scarcer the available assets. For example: For liabilities with maturities longer than 30 years, it is very difficult to find appropriate assets in market for matching.

5. Continued

- Accurate projection of cash flow: Understanding of Interest rate sensitivity of liabilities depends on the ability to project cash-flows. The challenges for accurate projection include
 - Policyholders' assumption can be deviated significantly from assumption due to lapses, withdrawals and mortality
 - Difficulties in calibration of parameters for stochastic scenarios.
 - Difficulties in determining proper discount rate for very long horizons (beyond 30 years)
 - Inflation rate over long horizons can differ significantly from expected
- (b) Explain why the traditional duration measure (expressed as the present value of the weighted average time to the cash flows), is not appropriate for TJK.

Commentary on Question:

Candidates need to identify optionality of the products is the key reason that traditional duration measure is not appropriate. Credit would also be given if candidates identify effective duration as a good alternative for liability valuation.

The duration of a financial instrument or liability cash flow can only be determined if the future cash flows are known and are not interest sensitive. For liability cash flows on insurance products or pension liabilities, optionality exists which implies cash flows are interest sensitive. In this case, effective duration would be a better alternative.

- (c) Assess this strategy with respect to:
 - (i) Interest rate risk
 - (ii) Liquidity risk

Commentary on Question:

For ci, the general performance is not very well. Many candidates fail to see both sides of the impact of NFI to interest rate risk; while NFI creates mismatch risk, it can also help mitigate interest rate risk due to its low correlation with interest rate fluctuation.

For cii, many candidates identify NFI as illiquid assets but fail to elaborate further and explain why these assets are illiquid. Full credit will only be given to candidates who identify TJK needs to address the liquidity requirements of liabilities.

5. Continued

ci)

Incorporating private equity or real estate can mitigate interest rate sensitivity relative to public bonds. These non-fixed-income (NFI) assets generally lack a direct correlation with interest rate fluctuations; however, they may offer alternative return sources through rental income (real estate) or business cash flows (private equity).

However, a mismatch risk persists if these assets are employed to support liabilities sensitive to interest rates. This risk is especially pronounced when NFI assets are utilised within a carve-out strategy to back long-term liability cash flows.

cii)

Both private equity and real estate are illiquid assets. Incorporating them elevates the liquidity risk within the portfolio, particularly during stress scenarios or in the event of unforeseen liabilities.

Liquidity risk escalates when liabilities are redeemable on short notice while assets remain illiquid.

TJK is required to assess whether Non-Financial Investment (NFI) assets sufficiently address the potential liquidity requirements of liabilities, especially considering policyholder behaviours that could precipitate sudden cash outflows.

- (d) Recommend an approach to address any challenges identified in Part (c).

Commentary on Question:

This question is generally answered well. However, it needs to be noted that using derivatives as a tool to address liquidity risk is not an accepted answer. It is because derivatives possess counterparty risk and might not be easily monetized during a stress situation. As a result, derivatives are not considered highly liquid assets for addressing liquidity risk.

Interest Rate Risk: (One of the following approaches)

- Employ interest rate derivatives, such as swaps or bond futures, to synthetically replicate the interest rate exposure of liabilities.
- Integrate real estate or private equity with longer-duration bonds to compensate for the absence of duration in the new assets.
- Utilise models to simulate scenarios where interest rate risk might induce asset-liability mismatches and accordingly implement pre-hedging strategies.

Liquidity Risk: (One of the following approaches)

- Allocate illiquid assets exclusively to long-duration liabilities with predictable cash flows. Employ a “carve-out” or “surplus” portfolio strategy.
- Maintain a reserve of unencumbered high-quality liquid assets to satisfy short- and medium-term obligations.

5. Continued

- Conduct regular liquidity stress testing predicated on both historical and forward-looking scenarios, such as redemption surges and market freezes.
 - Formulate a liquidity plan that delineates trigger points, funding sources, and a hierarchy of liquidation.
- (e) Design a liquidity stress test for TJK.

Commentary on Question:

Candidates are required to identify and explain each major step in a stress test (Objective, Assumptions, Methodology, Governance & Response and Reporting) to receive full credit. Most candidates explained Assumptions and Methodology well, yet not many candidates identified the other 3 steps.

Objective Setting

To evaluate the liquidity adequacy of the plan's liquidity in the context of a concentrated retirement shock, wherein 20% of active participants retire and opt for a lump sum payout within a single year.

Assumptions Setting

It refers to the assumptions that need to be made for the stress test. For example,

- The percentage of active participants to the total population
- The average lumpsum payout for each retiree
- The liquidity source that is available (From liquid assets only such as cash and short-term fixed income)
- The market condition where the stress test is performed (Interest rate level, equity return, inflation rate level etc)

Methodology Setting

The necessary steps required to perform the stress test

Step 1: Estimate Total Lump Sum Payout (Total Liability)

Step 2: Identify Liquid Asset available before and after stress applied, need to consider the adjustment due to haircuts during stress

Step 3: Determine Liquidity Coverage ratio or Excess Liquidity of the scenario

Step 4: Perform Sensitivity testing or Reverse Stress Testing to understand the impact of stress on liquidity.

Governance and Response Plan Setting

Action to be taken after stress test is done

Liquidity Plan:

- Establish internal thresholds for triggering liquidity action
- Rank asset liquidation by least impact (e.g., sell government bonds before equities)

5. Continued

Communication to Senior Management:

- Notify the Seniors if any parameters deviate from baseline estimates.

Reporting Parameters

Key parameters need to be calculated for different scenarios to understand the impact on liquidity by stress

- Reporting parameters can be Liquidity Coverage Ratio or Capital Shortfall.

6. Learning Objectives:

3. The candidate will understand tools and strategies to manage ALM risks.

Learning Outcomes:

- (3c) Demonstrate an understanding of and apply tools and techniques for measuring and managing interest rate risk in an ALM context

Sources:

CP351-112-25: The Devil is in the Tails: Actuarial Mathematics and the Subprime Mortgage Crisis

Quantitative Enterprise Risk Management, Hardy, Mary and Saunders, David, 2022 Ch. 6 – Copulas

Commentary on Question:

This question tests candidates' understanding of the modeling of joint default behavior in collateralized debt obligations (CDOs). Candidates should in particular demonstrate their understanding of different copula models, including their characteristics and limitations. In general, candidates performed well except for the numerical calculations in part (b), which required an understanding of the relationship between individual bond losses and tranche losses.

Solution:

- (a) Your Chief Risk Officer makes the following statements:

- Since we invest only in senior tranches of CDOs, we are insulated from default risks of CDOs.
- Since the Li model is easy to understand, we should continue to use it to assess our risk exposures with CDOs.

Critique each statement.

Commentary on Question:

Candidates generally responded correctly to the first statement, noting problems with the statement; for the second, some candidates only focused on Li model's drawbacks, and, in particular, did not comment on its potential advantages.

Solutions

- (i) The CRO's statement is incorrect.

While senior tranche holders are less exposed to defaults compared to equity and mezzanine tranches, they are not completely insulated.

If enough losses occurred such that equity and mezzanine tranche holders no longer receive coupons, senior tranches begin to incur losses.

6. Continued

(ii) The CRO's statement is partially correct. While Li model has its own advantages including:

- Simple to understand,
- Enables fast computation
- Easy to calibrate since there is only one parameter, which is the pairwise correlation, that has to be estimated,

It also has a few drawbacks:

- It underestimates occurrence of joint defaults in the tail. As default size increases, company defaults become increasingly correlated as observed in the 2008 financial crisis.
- It does not give a consistent implied correlation for each tranche, which is not desirable for delta-hedging.
- It does not account for dynamic or time-varying model parameters, making it less robust under extreme scenarios.

- (b) Calculate the expected loss per year for each of the tranches, assuming defaults are independent between each underlying bond.

Commentary on Question:

Candidates performed poorly on this question. Very few answers demonstrated an understanding of the CDO working mechanism and therefore were able to set up the expected loss formula for each tranche.

Some candidates also had difficulties calculating the probabilities of each loss scenario.

Solutions

Equity tranche:

$$\text{Probability of no defaults} = (1 - 0.02)^{100} = 0.1326$$

$$\text{Probability of exactly 1 default} = 100 * 0.02 * (1 - 0.02)^{99} = 0.2707$$

$$\text{Probability of exactly 2 defaults} = \binom{100}{2} \times 0.02^2 \times (1 - 0.02)^{98} = 0.2734$$

$$\text{Probability of more than 2 defaults} = 1 - 0.1326 - 0.2707 - 0.2734 = 0.3233$$

$$\text{Expected loss on equity tranche} = 1 * 0.2707 + 2 * 0.2734 + 3 * 0.3233 = 1.7874$$

Mezzanine tranche:

$$\text{Probability of 3 defaults} = \binom{100}{3} \times 0.02^3 \times (1 - 0.02)^{97} = 0.1823$$

$$\text{Probability of 4 defaults} = \binom{100}{4} \times 0.02^4 \times (1 - 0.02)^{96} = 0.0902$$

$$\text{Probability of 5 defaults} = \binom{100}{5} \times 0.02^5 \times (1 - 0.02)^{95} = 0.0353$$

$$\text{Probability of more than 5 defaults} = 0.3233 - 0.1823 - 0.0902 - 0.0353 = 0.0155$$

6. Continued

*Expected loss on mezzanine tranche = $1 * 0.0902 + 2 * 0.0353 + 3 * 0.0155 = 0.2074$*

Senior tranche:

Since the total expected loss on the whole CDO = $100 \times 0.02 = 2$, the expected loss on the senior tranche is = $2 - 1.7874 - 0.2074 = 0.005224$

Alternatively, it can be calculated directly by summing over losses weighted by the probabilities of 6 or more defaults as $\sum_{k=1}^{94} k \times \binom{100}{k+6} \times 0.02^{k+6} \times (1 - 0.02)^{94-k} = 0.005224$

- (c) Explain the impact of an increase in the pairwise correlation on the expected loss for each tranche in the two plots above.

Commentary on Question:

Most candidates were able to identify the correct relationships between pairwise correlation and expected loss of each tranche. However, the explanation for such relationship was often absent or incorrect.

Solutions

Equity Tranche:

- *As correlation increases, the expected loss on the equity tranche decreases.*
- *This happens because higher correlation makes it more likely that either many bonds default or very few bonds default.*
- *Since the equity tranche absorbs first losses, an increase in the probability that few defaults occur reduces expected equity tranche losses.*

Mezzanine Tranche:

- *The effect of increased correlation on the mezzanine tranche is uncertain and depends on the probability of default.*
- *In some cases, the expected loss increases with correlation, while in others, it decreases.*

Senior Tranche:

- *As correlation increases, the expected loss on the senior tranche increases.*
- *This occurs because higher correlation increases the likelihood of extreme events where many assets default at the same time, surpassing the mezzanine tranche's limit of loss absorption.*

6. Continued

- (d) Identify the copula associated with the model in each plot. Justify your answer.

Commentary on Question:

This question was answered well by candidates, with candidates receiving full credit.

Solutions

Model A: Gaussian, no upper or tail dependence

Model B: Gumbel, exhibits upper tail dependence

Model C: Clayton, exhibits lower tail dependence

- (e) Recommend which of the three copulas above should be used to address the concern.

Commentary on Question:

This question was well answered by candidates, although a few candidates misidentified extreme loss scenarios as lower tail dependency.

Solutions

Model B (Gumbel copula) is more suitable to address the concern of the firm for modeling extreme loss scenarios, due to its ability to capture upper tail dependency where large default losses cluster.

7. Learning Objectives:

3. The candidate will understand tools and strategies to manage ALM risks.

Learning Outcomes:

(3d) Understand and evaluate model and parameter risks.

(3e) Explain and implement techniques used to mitigate market risks.

Sources:

CP351-113-25: Chapter 4 of ALM for Banks and Insurance Companies, Habart (excluding 4.3.6)

Quantitative Enterprise Risk Management, Hardy, Mary and Saunders, David, 2022 Ch. 15 – Risk Mitigation Using Options and Derivatives

Commentary on Question:

Commentary listed underneath question component.

Solution:

(a) Your colleague made the following statements:

- Implied volatility is better than historical volatility for projecting future returns because it reflects the market's expectations for the future.
- Correlations between equities can be ignored because they are difficult to estimate and often unstable.
- A global dividend yield should be used for simplicity and consistency.

Critique the statements above.

Commentary on Question:

Most candidates focused on either implied volatility or historical volatility and therefore did not provide a complete comparison between the two.

Regarding correlations, most candidates acknowledged that they should not be ignored but provided weak or vague explanations. Statements such as "correlations are important for future projections" were too general to obtain full credit.

Most candidates did not support the use of a global dividend yield and were able to point out that equities in different regions, sectors or asset classes may not have a homogeneous dividend yield.

7. Continued

Historical volatility should be used instead of implied volatility. This is because projecting future returns should be done under a real-world measure rather than a risk-neutral measure. Implied volatility is a parameter specific to the Black–Scholes model and is mainly used for market-consistent valuation, so using it outside this context may lead to miscalibrated risk estimates. Historical volatility, possibly combined with expert judgment or economic forecasts, is more appropriate for modelling future returns in ALM.

It is a valid concern that the correlation between two equities is not a very robust statistic and may depend heavily on the historical data used, but it should not be ignored completely. Such correlations can be statistically significant and are a key input for scenario generation.

A global dividend yield may not be appropriate. Some equities pay dividends, some do not, and others reinvest earnings. Additionally, equities in different regions are subject to different economic outlooks and competitive landscapes, which in turn affect dividend policies. Therefore, a single global dividend yield may misrepresent the actual cash flows of the portfolio unless dividend behavior is broadly similar across assets.

- (b)
- (i) Identify a strategy to achieve the payoff described in the graph above.
 - (ii) Calculate the cost of establishing this strategy using the Black–Scholes model.

Commentary on Question:

Candidates performed well on both parts.

For (i), full marks were awarded for correctly identifying the long/short position of the options and their strikes. Partial credits were awarded for flipped long/short positions and/or incorrect strikes.

For (ii), partial credits were awarded for minor calculation errors in the Black–Scholes formula.

7. Continued

- (i) Collar strategy: long a put option at 90, and short a call at 130.
 - (ii) Please refer to the accompanying Excel sheet for full calculations.
 $T = 1, K_{\text{put}} = 90, K_{\text{call}} = 130$. Using the Black–Scholes formula, we can obtain price of put = 4.31, and price of call = 2.66. The cost of the strategy is then $4.31 - 2.66 = 1.65$.
- (c) Calculate the 1-year 95% Value-at-Risk for:
- (i) The unprotected portfolio, assuming that the assets are fully invested in the ETF.
 - (ii) The protected portfolio with the strategy identified in Part (b)(i).

Commentary on Question:

Value-at-Risk (VaR) represents the potential maximum loss amount. A common issue here is that some candidates reported the value of the portfolio instead of the loss amount.

For part (i), as the distribution of returns is known, the exact amount of VaR can be determined. The delta-normal approach is an approximation method used when the true distribution of the portfolio is unknown; applying it here would overstate the VaR. Partial credits were awarded to candidates who used the delta-normal approach.

For part (ii), candidates who understood the payoff structure of a collar could read the solution directly from the payoff diagram, and no additional calculations were required.

7. Continued

- (i) Please refer to the accompanying Excel sheet for full calculations. The log return follows a normal distribution, so the log return at 95% confidence level is

$$\mu - \Phi(0.95) \times \sigma = 6\% - 1.645 \times 25\% = -35.12\%$$

$$\text{ETF price at 95\% confidence level: } V_1 = 100e^{-35.12\%} = 70.38$$

$$95\% \text{ VaR: } L = V_0 - V_1 = 100 - 70.38 = 29.62$$

- (ii) When the price of the ETF is 70.38, one can exercise the put option and sell the ETF for 90. Therefore, the loss is capped at 10.
- (d) Calculate (use the “Goal Seek” function as needed) the new strike price of the call option in the strategy in Part (b)(i) so that the hedging cost is 0.

Commentary on Question:

Many candidates performed well on this part. To obtain full credit, candidates needed to ensure that the new call option was priced to match the put option in part (b)(ii), and to determine the corresponding call strike price that satisfies this condition. Common errors included matching the call option price to an incorrect value, or incorrectly reusing the original d_1 and d_2 values when deriving the new strike price.

Please refer to the accompanying Excel sheet for full calculations.

Use a similar setup to part (b)(ii). Adjust the strike price of the call option until its price matches that of the put option.