SOCIETY OF ACTUARIES

Quantitative Finance

Exam INV 201

Date: Monday, March 23, 2026

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has 7 questions numbered 1 through 7 with a total of 50 points.

   The points for each question are indicated at the beginning of the question.

2. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions provided in this document.

3. Each question part or subpart should be answered either in the Excel document or the paper provided as directed. Graders will only look at the work as indicated.

4. In the Excel document, answers should be entered in the box marked ANSWER. The box will expand as lines of text are added. There is no need to use special characters or subscripts (though they may be used). For example, β_1 can be typed as beta_1 (and ^ used to indicate a superscript).

5. Prior to uploading your Word and Excel files, each file should be saved and renamed with your unique candidate number in the filename. To maintain anonymity, please refrain from using your name and instead use your candidate number.

6. The Excel file that contain your answers must be uploaded before the five-minute upload period expires.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.

5. When you finish, hand in all your written-answer sheets to the Prometric center staff. Be sure to hand in all your answer sheets because they cannot be accepted later.

The responses for all parts of this question are required on the paper provided to you.

1.

(*6 points*) You are on the derivatives trading desk within a hedge fund.

The firm currently has an equity portfolio worth $100 million, with a beta of 1.0 relative to a stock market index. The index is currently at 5000. There are no dividend payouts for the index. Risk free rate is 2%.

The portfolio manager wants to perfectly hedge against the value of the portfolio dropping below $92 million in the next 1-year period.

(a) (*1 point*) Construct an options strategy that will achieve the portfolio manager's objective.

(b) (*2 points*)

(i) (*1 point*) Construct an options strategy if the beta of the portfolio increases from 1.0 to 2.0.

(ii) (*1 point*) Explain whether the cost of hedging will increase, decrease or not change under this new strategy.

Your analyst provides you with the following option prices on the index. All options have a 1-year remaining term to maturity.

Strike	Call (European)	Put (European)	Call (American)	Put (American)
4700	639.57	200.67	NA	208.98
4800	579.37	237.51	581.62	NA
4900	523.07	278.35	NA	290.05
5000	470.67	322.90	NA	320.79

Assume accuracy of the European option prices given.

(c) (*1 point*) Identify two issues in the American option prices provided in the table.

1. Continued

Two economists offered their views on the prospects of the index. Their responses differ and are summarized below:

- Economist A: The index will be above 4200 at the end of the next 1-year period.
- Economist B: The index will remain above 4200 throughout the next 1-year period.

(d) (*1 point*) Using European and American option(s):

(i) (*0.5 points*) Recommend the most cost-efficient hedge strategy if the view of economist A is accepted.

(ii) (*0.5 points*) Recommend the most cost-efficient hedge strategy if the view of economist B is accepted.

Assume the view of economist A is accepted.

(e) (*1 point*) Assess whether the cost of hedging would increase, decrease, or not change under an instantaneous increase in the volatility of the index.

The responses for all parts of this question are required on the paper provided to you.

2.

(*6 points*) Let $\{S_t\colon 0 \le t \le T\}$ be a risky asset with initial value $S_0 > 0$ whose price process evolves discretely. From any given time t, S_t moves up with probability p by a multiplicative factor of $\mu > 2$ and down by a multiplicative factor of $\frac{1}{\mu}$. Interest compounds discretely at the fixed rate of $r > 0$.

(a) (*2 points*)

(i) (*1 point*) Calculate the value of p such that the discounted price-process of S is a martingale by evaluating the expectation of $\frac{1}{(1+r)^{t+1}} S_{t+1}$,

(ii) (*1 point*) Verify your result using the definition of a martingale.

(b) (*1 point*) Derive the condition for there to be no arbitrage opportunity using the result from part (a).

You are given an equivalent probability measure to p where the probability of an up-movement is given by $\tilde{p} = \frac{1}{\mu - 1}$.

(c) (*1 point*)

(i) (*0.5 points*) State whether $\frac{1}{(1+r)^{t+1}} S_{t+1}$ is a martingale with respect to $\tilde{p}$.

(ii) (*0.5 points*) Derive the value for the time *t+1* Radon-Nikodym derivative process, as measured at time t, with respect to the probability measures with corresponding “up” probabilities of p and $\tilde{p}$.

(d) (*2 points*)

(i) (*1 point*) Calculate today’s price of the option whose payoff in terms of S_0 is given as $\sqrt{S_2}$ if $\mu = 3, r = 0.05$.

(ii) (*1 point*) Verify your result under the $\tilde{p}$ measure by direct calculation using part (c) (ii) above.

The responses for all parts of this question are required on the paper provided to you.

3.

(*8 points*) You are an actuarial assistant working in the variable annuity hedging department of a large life insurance company. Your manager would like you to investigate stochastic models for equity indices. Let $(B_t)_{t\geq 0}$ be a standard Wiener process. In what follows, you may use the Ito Isometry Theorem, given by the formula:

$$E\left[\left(\int f(t,B_t)\,dB_t\right)\left(\int g(t,B_t)\,dB_t\right)\right] = E\left[\int f(t,B_t)\,g(t,B_t)\,dt\right].$$

(a) (5 *points*)

(i) (*1 point*) Verify that $B_1^2 = \int_0^1 2B_t\,dB_t + 1$.

(ii) (*1.5 points*) Verify that $Cov\left(B_1^2, \int_0^1 e^{-t}B_t\,dB_t\right) = 2 - 4e^{-1}$.

(iii) (*2.5 points*) Derive $Var\left[\int_0^1 e^{-t}B_t^2 dt\right]$.

Assume $(S_t)_{t\geq 0}$ is an equity index price process. Let μ and σ be positive constants.

(b) (*1 point*) Critique the following stochastic model for the equity index:

$$dS_t = \mu\,dt + \sigma\,dB_t.$$

Now assume the following stochastic model for the equity index:

$$dS_t = \mu S_t\,dt + \sigma S_t\,dB_t.$$

Your manager makes the following statement:

> "If the equity index follows the above model, then its expected continuously compounded return per annum realized over a period $[0, T]$ must be less than μ."

(c) (*1 point*) Evaluate your manager's statement.

Moreover, suppose that the current index price is 5750, the estimated value for σ is 20%, and the estimated value for μ is 10%.

(d) (*1 point*) Calculate the 95% confidence interval of the index price in 6 months.

The responses for some parts of this question are required on the paper provided to you.

4.

(*6 points*) You are interested in using the Vasicek model for daily three-month (annualized, continuously compounded) interest rates over a 5-year period. However, you are concerned that the Vasicek model may yield negative interest rates.

(a) (*1 point*) Derive an expression for the probability $P[r_{t+s} < 0|r_t]$, where r_{t+s} and r_t are the interest rates at times $t + s$ and t in years, respectively, with $s > 0$ and $t \geq 0$.

Given the parameters $\gamma = 0.5, \bar{r} = 0.04, \sigma = 0.02$ and $r_t = 0.03$.

(b) (*2 points*) Create a chart by plotting the values of $\mathrm{P}[r_{t+s} < 0|r_t = 0.03]$ for $s = 0.1, 0.2, \dots, 5$.

The response for this part is to be provided in the Excel spreadsheet.

Assume that the maximum value of the probability $\mathrm{P}[r_{t+s} < 0|r_t]$ in the chart occurs at $s = \bar{s}$.

You are given 100 standard normal random numbers the Excel spreadsheet of this question.

(c) (*2 points*) Approximate the probability $\mathrm{P}[r_{t+\bar{s}} < 0|r_t = 0.03]$ using the simulation.

The response for this part is to be provided in the Excel spreadsheet.

(d) (*1 point*) Compare the two methods in Parts (b) and (c) used to calculate the probability $\mathrm{P}[r_{t+\bar{s}} < 0|r_t = 0.03]$

The responses for all parts of this question are required on the paper provided to you.

5.

(*7 points*) A trader holds a portfolio of Bermudan put options on a non-dividend-paying stock

- Spot price S = 100
- Risk-free rate r = 3%, continuously compounding
- Volatility σ = 20%

Each Bermudan put option has the following properties:

Exotic Option	**Strike**	**Early Exercise Date**	**Maturity**	**Delta**	**Gamma**	**Vega**
Bermudan Put	115	Quarter-end	1 year	-0.7	0.025	25

The position (long/short) and quantity (number of contracts) for the Bermudan put are as follows:

Exotic Option	**Position**	**Quantity**
Bermudan Put	Long	100

For the Bermudan put option, the trader wants to value it using a binominal tree with a quarterly time step (Δt=0.25).

(a) (*3 points*)

(i) (*0.5 points*) Calculate the up and down factors (*u, d*).

(ii) (*0.5 points*) Calculate the risk-neutral probability (*p*).

(iii) (*0.5 points*) Sketch the binominal tree by showing the tree nodes with proper notations based on the calculations done in part (i) and part (ii).

(iv) (*0.5 points*) Describe how to value the option using the binominal tree.

(v) (*1 point*) Describe how to estimate the option delta and gamma.

5. **Continued**

Now assume that the trader added the following two types of options to his portfolio:

- Up-and-in Barrier Call
- Geometric Average Asian Call

Exotic options' strikes, barriers, and maturities as well as the Greeks per contract, are as follows:

Exotic Option	Strike	Barrier	Maturity	Delta	Gamma	Vega
Up-and-in Barrier Call	110	115	6 months	0.06	0.01	40
Geometric Average Asian Call	110	N/A	6 months	0.15	0.03	9.5

The positions (long/short) and quantities (number of contracts) for the call options are as follows:

Exotic Option	Position	Quantity
Up-and-in Barrier Call	Short	100
Geometric Average Asian Call	Short	100

(b) (*1 point*) Calculate the total portfolio delta, gamma, and Vega, considering the quantities and positions (short/long).

5. Continued

After adding these two types of call options, the trader becomes concerned about the risk to which the exotic option portfolio is exposed and decides to use the underlying stock shares and the following vanilla European put and call options to neutralize the portfolio delta, gamma, and Vega.

Vanilla Option	**Option Value**	**Strike**	**Maturity**	**Delta**	**Gamma**	**Vega**
Put	2.80	90	1 year	-0.22	0.015	30
Call	5.30	110	1 year	0.41	0.02	40

Assume that you cannot trade fractional option contracts, nor fractional stock shares.

Given the available hedging instruments and constraints:

(c) (*3 points*)

(i) (*1.5 points*) Describe a reasonable strategy to neutralize the portfolio delta, gamma, and Vega.

(ii) (*1.5 points*) Calculate the positions to be traded to implement the strategy you described above. Justify your answer.

The responses for all parts of this question are required on the paper provided to you.

6.

(*8 points*) An option arbitrageur considers buying a 1-year European call option on a non-dividend-paying stock with the following parameters:

- Stock price today (S) = \$100
- Strike price (K) = \$100
- Risk-free rate (r) = 5%, continuously compounding
- Implied volatility (σ_i) = 20%
- Actual volatility (σ_a) = 30%

The arbitrageur's strategy is to buy the call option in the open market and then delta-hedge discretely with implied volatility using the Black-Scholes-Merton model till the option expiration.

(a) (*1 point*) Explain why the arbitrageur expects to make a profit when hedging with implied volatility.

For part (b) only, assume the stock price stays near the strike until the option expiration.

(b) (*2 points*) Approximate the profit from this strategy.

Now suppose the arbitrageur considers hedging with an intermediate volatility ($\sigma_h =$ 25%) that is between the implied volatility (σ_i = 20%) and the actual volatility (σ_a = 30%).

(c) (*3 points*)

(i) (*1.5 points*) Compare hedging with σ_h, σ_a, and σ_i.

(ii) (*1.5 points*) Calculate the minimum and maximum possible profit if the arbitrageur hedges with $\sigma_h = 25\%$.

Now suppose that the arbitrageur extends the strategy to hold a portfolio of options, puts and calls on the same underlying stock but with different strikes and different expirations.

(d) (*1 point*) Describe how to find the expected profit at the portfolio level when hedging with implied volatilities.

(e) (*1 point*) Assess whether diversification across strikes reduces the uncertainty of the expected profit.

The responses for some parts of this question are required on the paper provided to you.

7.

(*9 points*) A large life insurance company ABC sells both traditional variable annuities (TVA) and Registered Index-Linked Annuities (RILA). More recently, the company is facing a major change in GAAP requirements, known as LDTI (Targeted Improvements to the Accounting for Long-Duration Contracts).

(a) (*1.5 points*)

(i) (*1 point*) Explain the differences between TVA and RILA in in terms of Basis risk.

(ii) (*0.5 points*) Explain why hedging is more effective for RILA than for TVA.

The following assumptions are for part (b) and (c)

During early years, the company offered GMAB contracts with a ROP feature on the benefit base. The following assumptions are defined:

- The initial guaranteed amount is G_0. The contract will mature in T_1 years.
- F_t is the value of the fund at time t ($t < T_1$). A fee of m is charged continuously on the fund value while the contract is in-force.
- F_t is linked to the equity index as $F_t = F_0 \frac{S_t}{S_0}$, where the underlying asset S_t follows risk-neutral SDE process of $dS_t = rS_t dt + \sigma S_t dW_t$, ($r$ is the constant risk-free rate, σ is the constant volatility of S_t).
- The probability of the insured with age x surviving until time t is given by ${}_tP_x$. The mortality probability is assumed independent of the fund return distribution. The force of mortality is $\mu_{x+t} = -\frac{d\ {}_tPx/dt}{{}_tPx}$.
- The time-0 PV of gross benefit payment G_{T_1} at maturity is given by $PV(G_{T_1}, t = 0) = {}_{T_1}P_x\, E\left[e^{-rT_1} max\,(G_0,\ F_{T_1})\right]$, where $G_{T_1} = max\,(G_0,\ F_{T_1})$

(b) (*1.5 points*) Write down the no-arbitrage time-0 price of the GMAB gross benefit payment, $PV(G_{T_1}, t = 0)$, in the form of Black-Scholes formula using the variables defined in the assumptions above and the d1, d2 functions below:

$$d_1(T, x) = \frac{\ln(x) + (r - m + \sigma^2/2)T}{\sigma\sqrt{T}}$$
$$d_2(T, x) = d_1(T, x) - \sigma\sqrt{T}$$

7. Continued

A few years after the GMAB policies were issued, the company rolls out a new GMDB rider product which offers a ROP feature. The new GMDB contract matures at T_2. The company allows the in-force GMAB policyholders to enter a new GMDB policy when the current GMAB contracts mature at T_1, and the accumulated benefit from the GMAB will be the initial benefit base for the new GMDB policy (is not deducted from the account value). The pricing department of the insurer is to determine the rider fee for the GMDB rider.

The following two quantities are defined:

m - The nominal annualized fee rate that will be deducted from the account value
m_d –The rider charge of the new GMDB policy (is not deducted from the account value)

(c) (*3 points*)

(i) (*2 points*) Write down the time-0 value of GMAB benefit policy that is currently in-force and will enter a GMDB policy upon maturity (denoted as $B_{GMAB,then\ GMDB}(t, F_t)$).

(ii) (*1 point*) Derive the formula of the GMDB rider charge, m_d, under the no-arbitrage assumption.

Hint: The benefit payout of a GMAB policy that is in-force and later enters a GMDB policy is $max\ (G_0, F_{T_1})(1 - \frac{F_s}{G_{T_1}})_+$, $where\ s\ =\ time\ of\ death$ and $(1 - \frac{F_s}{G_{T_1}})_+ = max(0,1 - \frac{F_s}{G_{T_1}})$

7. Continued

With the new LDTI regulation changes, the company has pivoted its focus to RILA products that requires less reserve capital. Given the following pricing assumptions and RILA product features:

RILA product feature or pricing assumption	Value
Underlying Asset – Current Price (S_0)	100
Dividend Yield (q)	0%
Implied Volatility (σ)	15%
Term in years (t)	3
Risk-Free Rate (r)	4%
Buffer (The first portion of the loss of investment in percentage that will be absorbed by the insurer)	10%
Participation rate	100%

The RILA deposit is invested in a zero-coupon bond and a portfolio of three European options. The portfolio of options consists of a long position of an ATM call option, a short position of an OTM call option and a short position of an OTM put option.

(d) (*2 points*) Calculate the fair cap rate at which the present value of the RILA product equals the initial premium.

The response for this part is to be provided in the Excel spreadsheet.

The company anticipates an economic downturn in the future that will cause the index volatility to move from 15% to 18%, which will reduce the option cost (value of the above three European options).

(e) (*1 point*) Explain how it could impact the calculated cap rate.

****END OF EXAMINATION****