SOCIETY OF ACTUARIES

**Asset Liability Management**

# Exam CP-351

**Date:** Friday, March 27, 2026

## INSTRUCTIONS TO CANDIDATES

### General Instructions

1. This examination has 7 questions, numbered 1 through 7, with a total of 50 points.

   The points for each question are indicated at the beginning of the question.

2. If a question asks for a recommendation, you must justify your answer

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions provided in this document.

### Written-Answer Instructions

1. Each question part or subpart should be answered either in the Word document or the Excel file as directed. Graders will only look at work in the indicated file.

   a) In the Word document, answers should be entered in the box marked ANSWER. The box will expand as lines of text are added. There is no need to use special characters or subscripts (though they may be used). For example, $\beta_1$ can be typed as beta_1 (and ^ used to indicate a superscript).

   b) In the Excel document, formulas should be entered. Performing calculations on scratch paper or with a calculator and then entering the answer in the cell will not earn full credit. Formatting of cells or rounding is not required for credit.

   c) For each question part requiring an answer in Excel, (1) clearly identify the inputs to the calculations, (2) show the necessary interim calculations, adding rows and / or columns, if necessary, and (3) enter the final answer in some or all of the cells highlighted in yellow, as applicable in each circumstance. These cells should contain formulas with links to other calculations in the worksheet. Minimize the use of hard-coded figures and maximize the number of interim steps in the calculations that would demonstrate your line of thinking.

2. The answer should be confined to the question as set.

3. Prior to uploading your Word and Excel files, each file should be saved and renamed with your unique candidate number in the filename. To maintain anonymity, please refrain from using your name and instead use your candidate number.

4. The Word and Excel files that contain your answers must be uploaded before the five-minute upload period expires.


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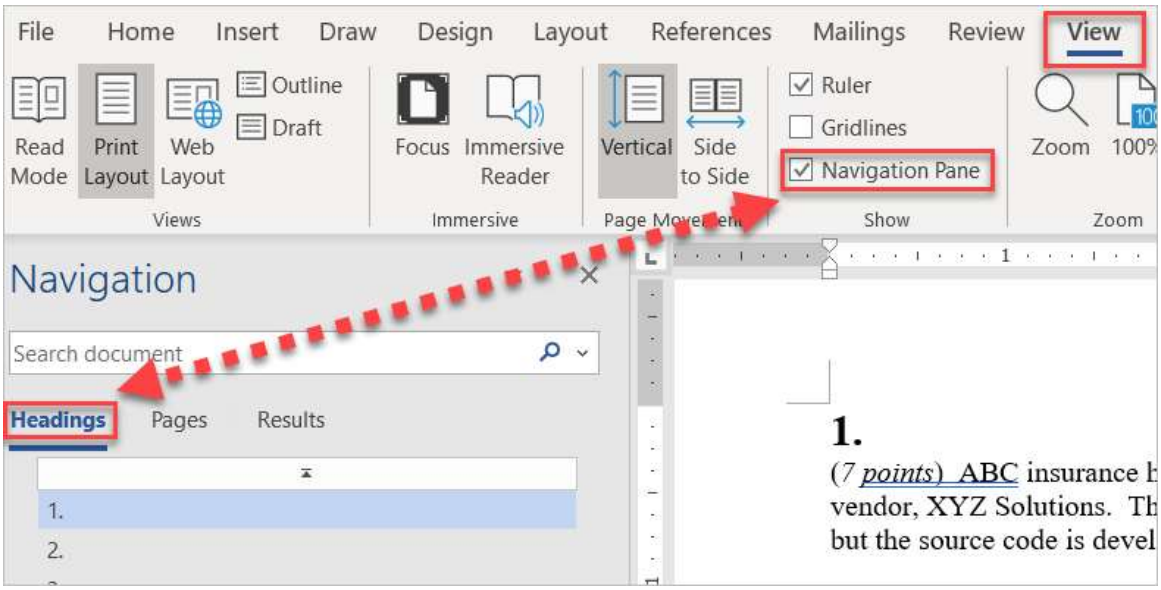
**Navigation Instructions**

Open the Navigation Pane to jump to questions.

Press Ctrl+F, or click View > Navigation Pane:



# 1.

(*6 points*) You are an actuary advising the trustees of DEF Pension Scheme, a defined benefit scheme with long-term liabilities. The scheme has faced funding volatility due to recent market fluctuations, prompting the trustees to explore a Liability Driven Investment (LDI) strategy.

(a) (*1 point*) Identify two main unrewarded risks that DEF Pension Scheme faces.

| ANSWER: |
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DEF is considering the following investments:

- Fixed rate bonds
- Inflation linked bonds
- Interest rate swaps

(b) (*2 points*) Explain how the three investment choices above can help DEF mitigate the unrewarded risks identified in Part (a).

| ANSWER: |
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You are given the following information about DEF Pension Scheme:

- Present value: £80 million
- Duration: 20 years

The trustees are considering two instruments to hedge the interest rate risk of the liability:

1. A 20-year zero-coupon bond
    - face value: £100 million
    - yield: 3%
2. An interest rate swap
    - PV01 per £1 million notional amount: £400

(c) (*1 point*) Calculate the PV01 of the zero-coupon bond.

| *The response for this part is to be provided in the Excel spreadsheet.* |
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## 1. Continued

The trustees decide to invest in the 20-year zero-coupon bond and hedge the remaining liability with the interest rate swap.

(d) (*2 points*) Calculate the notional amount of the swap to fully hedge the liability's PV01 along with the bond.

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| *The response for this part is to be provided in the Excel spreadsheet.* |

## 2.

(*7 points*) You are a consultant advising an insurance company whose liability duration has recently lengthened due to the introduction of a new line of deferred annuity products. The investment team has a strong preference for 10-year corporate bonds. However, its current ALM guidelines require the duration of the assets to match that of the liabilities, which is 20 years.

(a) (*1 point*) Recommend a structured investment approach using derivatives that allows the investment team to maintain exposure to the preferred 10-year corporate bonds while meeting the ALM guidelines.

| ANSWER: |
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(b) (*1 point*) Identify three conditions under which this strategy might backfire from a valuation or earning perspective, even if the duration matching is successful.

| ANSWER: |
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The insurance company has acquired a block of new business with the following two liability segments:

**Segment L1**:
Present Value (PV) = $400 million
Duration = 10 years

**Segment L2:**
PV = $300 million
Duration = 35 years

The company allocates two assets to support each segment of liability as follows:

**Asset A** for supporting **L1**:
PV = $450 million
Duration = 8 years

**Asset B** for supporting **L2**:
PV = $200 million
Duration = 40 years

## 2. Continued

(c) (*1 point*) Calculate the leverage ratio for each segment.

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| *The response for this part is to be provided in the Excel spreadsheet.* |

(d) (*1 point*) Assess whether each segment is under-hedged or over-hedged.

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| *The response for this part is to be provided in the Excel spreadsheet.* |

The new business contains a significant block of Canadian participating policies with liability cash flows extending beyond 30 years. Due to limited availability of long-duration instruments in the fixed income market, the ALM team has proposed a carve-out strategy replacing fixed income investments beyond 20 years with real estate and infrastructure investments.

(e) (*3 points*) Evaluate this proposal by addressing the followings:

(i) (*1 point*) Interest rate mismatch

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| ANSWER: |

(ii) (*1 point*) Liquidity and product design

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| ANSWER: |

(iii) (*1 point*) Capital implications under the Canadian Life Insurance Capital Adequacy Test (LICAT)

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| ANSWER: |

# 3.

(*8 points*) ABC sells long-term annuities backing pension obligations, which contain no policyholder options.

The liabilities have the following present values of projected cash flows (CF) based on various discount rates. The current liability discount rate is 6.5%.

| Discount rate | CF from years 1-15 | CF from years 16+ | Total |
|---|---|---|---|
| 5.00% | $ 31,500 | $ 3,685 | $ 35,185 |
| 5.25% | 31,100 | 3,505 | 34,605 |
| 5.50% | 30,700 | 3,330 | 34,030 |
| 6.50% | 29,275 | 2,760 | 32,035 |
| 6.55% | 29,150 | 2,725 | 31,875 |
| 6.60% | 29,075 | 2,695 | 31,770 |
| 6.80% | 28,800 | 2,585 | 31,385 |
| 7.00% | 28,525 | 2,490 | 31,015 |

The company uses a carve-out strategy to enhance the returns of its asset portfolio, which:

- Immunizes all liabilities due in the next 15 years with fixed income without utilizing any leverage.
- Uses equities and long-term fixed income with leverage to back the residual liabilities.

ABC has a target capital ratio of 150%. The required capital for assets held by ABC is as follows:

| | Required capital |
|---|---|
| **Public fixed income** | 0.5% |
| **Equity** | 20% |

(a) (*2 points*) Calculate the following for the assets in the immunizing portion of the portfolio:

(i) (*0.5 points*) Effective duration

*The response for this part is to be provided in the Excel spreadsheet.*

(ii) (*0.5 points*) Effective convexity

*The response for this part is to be provided in the Excel spreadsheet.*

**3.** **Continued**

(iii) (*1 point*) Required capital amount

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| *The response for this part is to be provided in the Excel spreadsheet.* |

The risk-free rate is 5% and the inflation rate has been historically 4% per year.
ABC is considering one of the two equity investments:

| Equity | Equity risk premium (Annualized) | Standard deviation $\sigma$ (Annualized) |
|---|---|---|
| Equity A | 10% | 20% |
| Equity B | 6% | 15% |

A portfolio manager has made the following comment: "If we assume that inflation rate would remain at 4%, I recommend Equity A over Equity B for the carve-out strategy."

(b) (*1 point*) Assess the portfolio manager's comment.

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| ANSWER: |

ABC has decided to use a mix of long-term bonds and $783 of equities for the carve-out portion.

Assume that:

- All leverage in the portfolio comes from assets backing required capital.
- Equity and interest rates are uncorrelated.
- The annualized standard deviations for the equity market and interest rates are given as follows:

| | Standard deviation $\sigma$ (Annualized) |
|---|---|
| **Equity market** | 20% |
| **Interest rates** | 0.50% |

## 3. Continued

You are also given the following information:

| Liability cash flow | Duration | Convexity |
|---|---|---|
| Years 1-15 | From part (a) | From part (a) |
| Years 16+ | 12 | 1,800 |

| Assets | Duration | Convexity |
|---|---|---|
| Long-term fixed income | 14 | 250 |
| Assets backing required capital | 2 | 10 |

Total risk is defined as the annualized standard deviation in economic surplus due to changes in both interest rates (not spreads) and equity.

You are given the following template for calculating the total risk of the portfolio.

**Interest Rate Risk**

| | Interest Rate Risk | | | | | Equity Risk | | |
|---|---|---|---|---|---|---|---|---|
| | Amount | Duration | Dollar Duration | Convexity | Dollar Convexity | Amount | Risk per Equity | Risk |
| Assets | | | | | | | | |
| Long-term Fixed Income | | | | | | | | |
| Equity | | | | | | | | |
| Required Capital for Long-term Fixed Income | | | | | | | | |
| Required Capital for Immunization Portfolio | | | | | | | | |
| Required Capital for Equity | | | | | | | | |
| Total Assets | | | | | | | | |
| Total Liabilities | | | | | | | | |
| Net Assets - Liabilities | | | | | | | | |

**Total Risk**

| | |
|---|---|
| Interst rate risk | |
| Equity risk | |
| Total Risk | |

(c) (*4 points*) Calculate the total risk of the portfolio.

*The response for this part is to be provided in the Excel spreadsheet.*

(d) (*1 point*) Calculate the total leverage ratio that the carve-out portion of the portfolio utilizes.

*The response for this part is to be provided in the Excel spreadsheet.*

# 4.

(*6 points*) CGS has a block of annuities. When calculating the effective duration of its liability, CGS assumes five key rate shifts, with each shift being a 20 bps upward shock to the key rate. The key rate durations for the first four key rate shifts are shown below:

| Key rate shift | Key rate duration |
|---|---|
| 1 | 1.2 |
| 2 | 2.1 |
| 3 | 1.7 |
| 4 | 4.5 |
| 5 | ? |

CGS's model output indicates that the fifth key rate shift would result in a 0.62% decrease to the value of the liability.

(a) (*1 point*) Calculate the effective duration of the liability.

*The response for this part is to be provided in the Excel spreadsheet.*

CGS's liability can be modeled by the function V(x):

$$V(x) = ax^2 + 4ax + b,$$

where a and b are constants, and x represents the current level of interest rate.

## 4. Continued

(b) (*1.5 points*) Write down a formula for the following in terms of V(x), a, b, and x:

(i) (*0.75 points*) Effective duration

| ANSWER: |
|---|

(ii) (*0.75 points*) Effective convexity

| ANSWER: |
|---|

Based on the current capital market environment, the company sets the value of x to 3%.

(c) (*1.5 points*) Calculate the convexity of the liability.

| *The response for this part is to be provided in the Excel spreadsheet.* |
|---|

CGS invests in Bond Y, which has the following cash flows:

| Year | Cash flow |
|---|---|
| 1 | $7 |
| 3 | $3 |
| 7 | $19 |

The current price of Bond Y is $20.

(d) (*2 points*) Calculate the yield-based duration of Bond Y. (Use the "Goal Seek" function as needed.)

| *The response for this part is to be provided in the Excel spreadsheet.* |
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# 5.

(*8 points*) You are an actuary at TJK overseeing the Asset-Liability Management (ALM) function of its defined benefit pension plan. Approximately, half of the participants are active (still working), and the other half are retired. Upon retirement, the active participants can choose either a lump sum payout or an immediate annuity. About 30% of the active participants are expected to retire in the next five years. Given the long-term nature of the liabilities and their exposure to economic risks, the Board of Directors has asked you to review different areas of TJK's ALM practice.

(a) (*1.5 points*) Describe three challenges TJK faces in ALM as it relates to long-term liabilities.

| ANSWER: |
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(b) (*1 point*) Explain why the traditional duration measure (expressed as the present value of the weighted average time to the cash flows), is not appropriate for TJK.

| ANSWER: |
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Currently, TJK only holds bonds and public equity in its asset portfolio. To enhance the yield, TJK is considering adding private equity or real estate to the asset portfolio.

(c) (*2 points*) Assess this strategy with respect to:

(i) (*1 point*) Interest rate risk

| ANSWER: |
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(ii) (*1 point*) Liquidity risk

| ANSWER: |
|---|
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(d) (*1.5 points*) Recommend an approach to address any challenges identified in Part (c).

| ANSWER: |
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## 5. Continued

TJK's audit committee requests management to conduct a liquidity street resting, assuming that 20% of the active participants are expected to retire and choose a lump sum payout next year.

(e) (*2 points*) Design a liquidity stress test for TJK.

| ANSWER: |
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# 6.

(*8 points*) You are an actuary working for a financial firm that actively invests in collateralized debt obligations (CDOs).

Your Chief Risk Officer makes the following statements:

- Since we invest only in senior tranches of CDOs, we are insulated from default risks of CDOs.
- Since the Li model is easy to understand, we should continue to use it to assess our risk exposures with CDOs.

(a) (*1.5 points*) Critique each statement.

| ANSWER: |
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Your firm has invested in a CDO with the following structure:

- It consists of 100 underlying 1-year bonds with equal notional values.
- Each bond pays a coupon of $1 million per year, which is redistributed to all CDO holders.
- The probability of default is 2% per year for each bond. Defaults are assumed to happen only on coupon payment dates.
- The loss given default percentage is 100%.
- The distribution of the CDO notional by tranches is as follows:

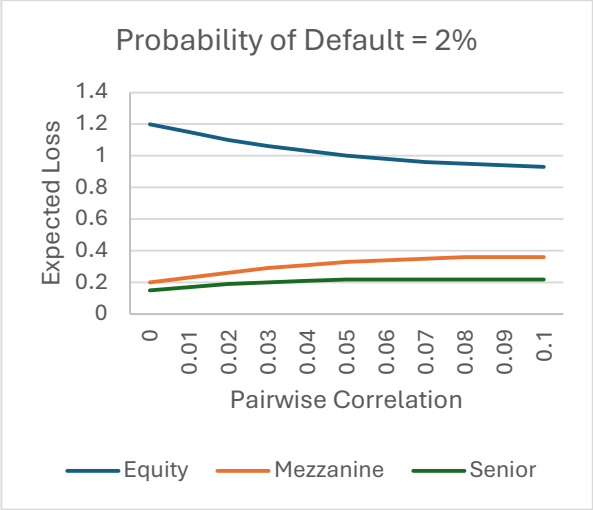
| **Tranche** | **Percentage of the CDO's notional** |
|---|---|
| Senior | 94% |
| Mezzanine | 3% |
| Equity | 3% |

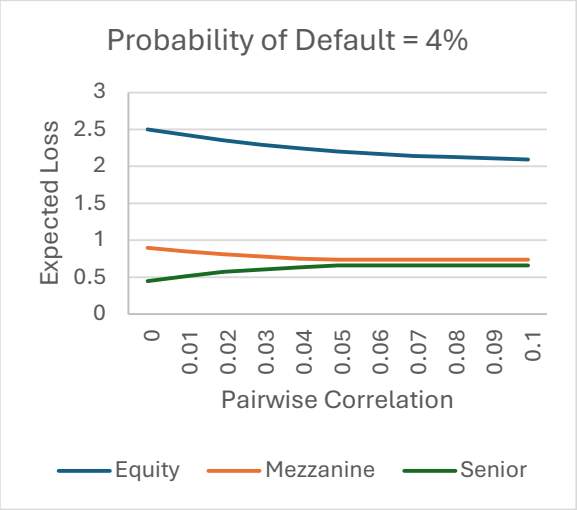
(b) (*3 points*) Calculate the expected loss per year for each of the tranches, assuming defaults are independent between each underlying bond.

| *The response for this part is to be provided in the Excel spreadsheet.* |
|---|

## 6. Continued

Your firm has invested in another CDO. The credit model assumes that the defaults of the underlying bonds are correlated and share the same pairwise correlation. The following plots present the expected loss of each tranche under different values of pairwise correlation, with probability of default for each bond being 2% and 4%, respectively:




(c) (*1.5 points*) Explain the impact of an increase in the pairwise correlation on the expected loss for each tranche in the two plots above.

| ANSWER: |
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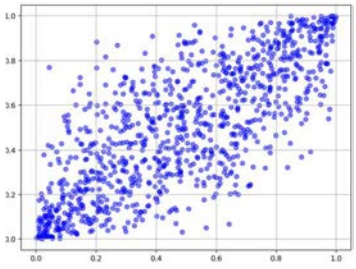
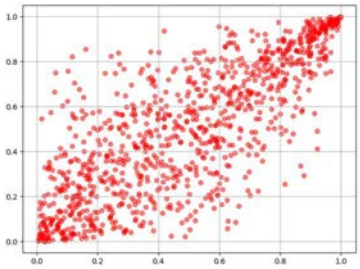
## 6. Continued

Your firm is assessing its approach for modeling loss dependency using three different copula models: Clayton, Gaussian and Gumbel. An analyst has generated the following scatterplots of simulated copula samples from two assets whose loss dependency is governed by these three models.

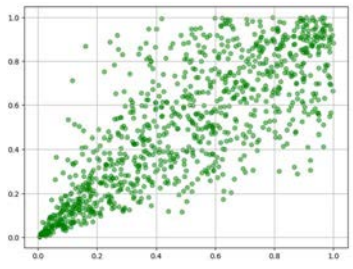
**Model A**



**Model B**



**Model C**



(d) (*1.5 points*) Identify the copula associated with the model in each plot. Justify your answer.

| ANSWER: |
|---|

Your firm is concerned about modelling the tail risk when multiple underlying assets default at the same time.

(e) (*0.5 points*) Recommend which of the three copulas above should be used to address the concern.

| ANSWER: |
|---|

## 7.

(*7 points*) You work for a large pension fund with broad investments across multiple asset classes and geographic regions. You and your colleague are reviewing the asset model used for evaluating the equity portfolios.

Your colleague made the following statements:

- Implied volatility is better than historical volatility for projecting future returns because it reflects the market's expectations for the future.
- Correlations between equities can be ignored because they are difficult to estimate and often unstable.
- A global dividend yield should be used for simplicity and consistency.

(a) (*2 points*) Critique the statements above.

| ANSWER: |
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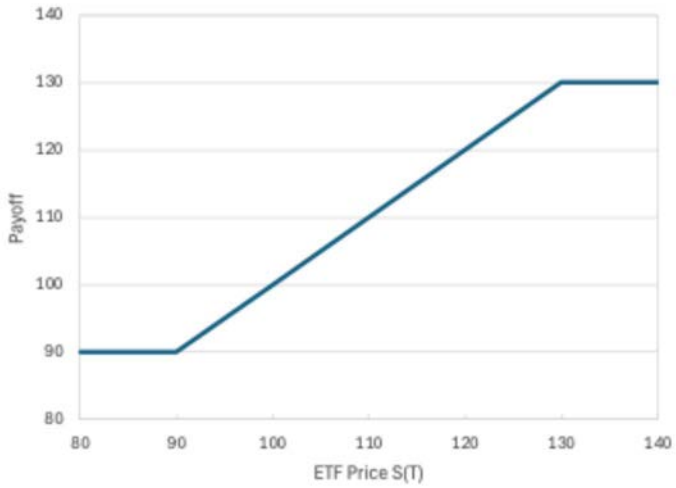
| |

## 7. Continued

A portfolio manager has invested a large portion of the assets in an ETF (Exchange Traded Fund) that tracks a market index. You are given the following information:

- The current price of the ETF is $S(0) = 100$.
- The return of the ETF follows a lognormal distribution with $\mu = 6\%$ and $\sigma = 25\%$.
- The risk-free rate $r$ is $3\%$.

The portfolio manager is aiming to construct a position using call and put options on this ETF that results in the following target payoff (including the ETF) in one year:




(b) (*2 points*)

(i) (*0.5 points*) Identify a strategy to achieve the payoff described in the graph above.

| ANSWER: |
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(ii) (*1.5 points*) Calculate the cost of establishing this strategy using the Black Scholes model.

| *The response for this part is to be provided in the Excel spreadsheet.* |
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## 7. Continued

(c) (*2 points*) Calculate the 1-year 95% Value-at-Risk for:

(i) (*1 point*) The unprotected portfolio, assuming that the assets are fully invested in the ETF.

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| *The response for this part is to be provided in the Excel spreadsheet.* |

(ii) (*1 point*) The protected portfolio with the strategy identified in Part (b)(i).

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|---|
| *The response for this part is to be provided in the Excel spreadsheet.* |

(d) (*1 point*) Calculate (use the "Goal Seek" function as needed) the new strike price of the call option in the strategy in Part (b)(i) so that the hedging cost is 0.

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| *The response for this part is to be provided in the Excel spreadsheet.* |

***END OF EXAMINATION****