SOCIETY OF ACTUARIES®

Advanced Short-Term Actuarial Mathematics Exam

Exam ASTAM

Date: Wednesday, April 22, 2026

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has 6 questions numbered 1 through 6 with a total of 60 points. The points for each question are indicated at the beginning of the question.

2. **Question 1 is to be answered in the Excel workbook. For this question, only the work in the Excel workbook will be graded.**

3. Questions 2-6 are to be answered in pen in the Yellow Answer Booklet provided. For these questions graders will only look at the work in the Yellow Answer Booklet. Excel may be used for calculations or for statistical functions, but any work in the Excel booklet will not be graded.

4. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions provided in this document.

Excel Answer Instructions

1. For Question 1, you should answer directly in the Excel Question worksheet. The question will indicate where to record your answers.
2. You should generally use formulas in Excel rather than entering solutions as hard coded numbers. This will aid graders in assigning appropriate credit for your work.
3. Graders for Question 1 will not have access to any comments or calculations provided in the Yellow Answer Booklet.
4. For Question 1, you may add notes to the Excel Question worksheet if you feel that might help graders. However, these should be entered directly into the Excel Question worksheet. Graders may not be able to read notes entered as comments.
5. When you finish, save your Excel workbook with a filename in the format xxxxx_ASTAM, where xxxxx is your candidate number. Your name must not appear in the filename.
6. Record your candidate number in the indicated cell in the Excel Question worksheet.

Pen and Paper Answer Instructions

1. Write your candidate number and the number of the question you are answering at the top of each sheet. Your name must not appear.

2. Start each question on a fresh sheet. You do not need to start each sub-part of a question on a new sheet.

3. Write in pen on the lined side of the answer sheet.

4. The answer should be confined to the question as set.

5. When you are asked to calculate, show all your work, including any applicable formulas, in the Yellow Answer Booklet.

6. If you use Excel for calculations for pen and paper answers, you should include as much information in the Yellow Answer Booklet as if you had used a calculator, including formulas and intermediate calculations where relevant. Written answers without sufficient support may not receive full or partial credit.

7. When you finish, hand in all your written answer sheets to the Prometric Center staff. Be sure to hand in all your answer sheets because they cannot be accepted later.

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BEGINNING OF EXAMINATION
ADVANCED SHORT-TERM ACTUARIAL MATHEMATICS

Provide the response for Question 1 in the Excel Question worksheet

1.

(*9 points*) You are given the following sample of claim frequency data from a study of 4,000 Property and Casualty insurance policies.

Claims, *k*	**Number of Policies,** n_k
0	3443
1	500
2	51
3	5
4	1
5+	0
Total	**4000**

(a) (*1.5 points*)

(i) Calculate the sample mean.

(ii) Calculate the sample variance.

(iii) State with reasons which of the $(a, b, 0)$ class of distributions could be fitted to these data.

1. Continued

(b) (*2 points*) You first fit the data to a geometric distribution with parameter β. You are given that the maximum likelihood estimate of β is the mean of the sample, as calculated in (a)(i) above.

In Table 1 below, p_k denotes $\Pr[N = k]$, where N is the claim frequency random variable, and l_k denotes the contribution to the log-likelihood from the contracts with k claims, assuming that N follows the geometric distribution, fitted by maximum likelihood.

(i) Complete Table 1.

(ii) Determine the maximum log-likelihood for the fitted geometric model.

Table 1: Geometric Likelihood			
Claims, k	**Number of Policies n_k**	p_k	l_k
0	3,443		
1	500		
2	51		
3	5		
4	1		
5+	0		
Total	**4,000**		

1. Continued

(c) (*2 points*) Next you fit a zero modified (ZM) geometric distribution to the data using maximum likelihood. Let ${}^{z}\beta$ denote the β parameter of the ZM geometric distribution, and let ${}^{z}p_k$ denote the probability function.

(i) Determine the MLE of ${}^{z}p_0$.

(ii) In Table 2 below, ${}^{z}l'_k$ denotes the derivative with respect to ${}^{z}\beta$ of the contribution to the log-likelihood from contracts with k claims. The derivative is evaluated using the value of ${}^{z}\beta$ given at the top of the table.

Table 2

${}^{z}\beta$ =	**0.1**	
Claims, k	Number of Policies n_k	${}^{z}l'_k$
0	3,443	0.000
1	500	-454.55
2	51	417.27
3	5	86.36
4	1	26.36
5+	0	0.00
Total	**4,000**	**75.45**

Use Goal Seek to determine the maximum likelihood estimate of ${}^{z}\beta$ for the zero modified geometric distribution. Use the table given to show your input for Goal Seek. You should find the MLE of ${}^{z}\beta$ is 0.115 to the nearest 0.001.

1. **Continued**

(d) (*2 points*) In Table 3, ${}^{z}l_k$ denotes the contribution to the log-likelihood from the contracts with k claims, assuming that N follows the ZM geometric distribution. Complete Table 3 using the ML estimates of ${}^{z}p_0$ and ${}^{z}\beta$. You should find that the maximum log-likelihood is –1820 to the nearest 10.

Table 3: ZM Geometric Likelihood			
Claims, k	**Number of Policies, n_k**	${}^{z}p_k$	${}^{z}l_k$
0	3,443		
1	500		
2	51		
3	5		
4	1		
5+	-		
Total	**4,000**		

(e) (*1.5 points*) Perform a likelihood ratio test to compare the fit of the geometric distribution to the ZM geometric distribution. Enter the requested information in the boxes provided.

2.

(*9 points*) NED Insurance Company sells a liability insurance policy. The policyholder can select a deductible of 0, 500, or 1000.

You have the following data from 2024.

Ground-Up Loss	Count	Total Ground-Up Losses
0-500	770	172,000
500-1000	450	328,000
1000-2000	520	748,000
2000-4000	450	1,276,000
4000-6000	140	687,000
6000+	170	1,914,000
Total	**2,500**	**5,125,000**

(a) (*2 points*)

(i) Calculate the expected payment per loss.

(ii) Calculate the expected insurance payment per loss with a deductible of 500.

(iii) Calculate the expected insurance payment per loss with a deductible of 1000.

NED develops their base rate using a deductible of 500.

(b) (*1 point*)

(i) Determine the indicated deductible relativity for a zero deductible policy.

(ii) Determine the indicated deductible relativity for a policy with a deductible of 1000.

2. Continued

NED is concerned about the tail risk for this portfolio.

(c) (*2 points*)

(i) Calculate the mean excess loss of the ground-up losses for a threshold of $x = 6000$.

(ii) State with reasons the implications of this result with respect to the tail of this distribution.

NED has requested a reinsurance company quote the cost of two different reinsurance contracts for this policy. The coverages are:

Contract A: excess of loss insurance with retention of 4000; that is, for each loss the reinsurer will pay any excess of the ground-up loss over 4000.

Contract B: pays 50% of the excess of the ground-up loss over 2000, with a maximum reinsurance payment of 2000.

(d) (*3 points*)

(i) Calculate the reinsurer's expected cost per loss for each of these reinsurance contracts.

(ii) NED is primarily concerned about tail risk for this policy. State with reasons which of these two reinsurance coverages is more appropriate.

(e) (*1 point*) NED is considering offering a policy with no deductible but with an upper limit of 6000. Describe one advantage and one disadvantage to the insurer of offering this policy, compared with a policy with no upper limit and with a 1000 deductible.

3.

(*10 points*) For a medical insurance policy, the number of claims follows a Poisson distribution with $\lambda = 2$. The amount of each claim follows an exponential distribution with $\theta = 1000$. Let S denote the aggregate claims random variable for an individual policy.

(a) (*2 points*)

(i) Calculate $E[S]$.

(ii) Calculate the standard deviation of S.

During 2025, the company expects to issue 10,000 independent policies. Let S_P denote the random variable representing the total claims for the portfolio.

(b) (*1 point*) Using the normal approximation, show that the 95th percentile of S_P is 20,330,000 to the nearest 10,000. You should calculate the value to the nearest 1000.

(c) (*2.5 points*) You are given the following assumptions used for calculating premiums.

- The total claims for the portfolio are assumed to be equal to the 95th percentile of S_P, as calculated in (b) above.
- Commissions are 8% of the gross premiums.
- Marginal expenses are 12% of the gross premiums.
- Fixed expenses during 2025 will be 300,000 to cover the 10,000 policies.
- The target profit margin, based on the above assumptions, is 5% of the gross premiums.

(i) Show that the gross premium rate per policy is 2,750 to the nearest 10. You should calculate the value to the nearest 1.

(ii) Calculate the expected profit per policy.

3. Continued

In order to do additional analysis, the company uses a discretized severity distribution. Two methods of discretizing the severity distribution are (i) the method of rounding (mass dispersal) and (ii) local moment matching.

(d) (*1.5 points*) List one advantage and one disadvantage of each method.

(e) (*3 points*) Let Y^d denote the discretized severity random variable, let $h = 400$ denote the discretization step size, and let $f_k = \Pr\left[Y^d = kh\right]$ for $k = 0, 1, 2, 3...$

(i) Calculate f_1 to 4 decimal places using the method of rounding.

(ii) Calculate f_1 to 4 decimal places using the method of local moment matching, using one moment.

4.

(*10 points*) Let *N* denote the claim frequency random variable for an automobile insurance policy. The distribution of *N* is assumed to depend on the driving habits of the policyholder. To model this, the insurer assumes that *N* depends on an unknown risk parameter Λ, where Λ varies by policyholder.

You are given that

- $N \mid \Lambda \sim$ Poisson(Λ)
- The prior distribution of Λ is a Gamma distribution, with parameters $\alpha = 2$ and $\theta = 0.2$.

(a) (*2 points*)

(i) Calculate $E[N]$

(ii) Calculate the standard deviation of *N*.

You are given the following claims history for an individual policyholder.\

Calendar Year	Policy Year	Number of Accidents
2023	1	0
2024	2	2
2025	3	0

(b) (*2 points*) Show that the posterior distribution of Λ is a Gamma distribution with parameters $\alpha^* = 4$ and $\theta^* = \frac{1}{8} = 0.125$.

(c) (*3 points*)

(i) Show that the predictive distribution of claim frequency for this policyholder is Negative Binomial.

(ii) Calculate E[*N*] under the predictive distribution.

4. Continued

(d) (*2 points*) Now suppose the claim frequency is estimated using Buhlmann credibility.

(i) Write down the values for the Buhlmann parameters, v, a, and μ.

(ii) Calculate the Buhlmann estimate of the claim frequency.

(e) (*1 point*) Your colleague suggests that the best estimate of the annual claim frequency for this policyholder is the estimate from the data, $\bar{X} = \frac{2}{3}$. Critique this suggestion.

5.

(*11 points*) You are analyzing the following cumulative claims run-off triangle for a commercial insurance. Assume that claims are fully developed by the end of Development Year 2.

	Development Year j		
Accident Year (AY) i	0	1	2
0	235	317	349
1	320	390	
2	416		

(a) (*2 points*) You first use the chain ladder method to estimate the ultimate claims.

(i) Show that the projected cumulative claims for AY1 are 430 to the nearest 10. You should calculate the value to the nearest 1.

(ii) Calculate the projected cumulative claims for AY2.

You next decide to estimate the outstanding claims using the Bühlmann-Straub model. You are given that $\hat{\gamma}_0 = 0.71303$, $\hat{\gamma}_1 = 0.19528$, and $\hat{\gamma}_2 = 0.09169$.

(b) (*1 point*) Explain in words what the γ_j values represent.

(c) (*3 points*) You are given that $s_0^2 = 625.41$ and $\overline{C} = 440.61$.
Show that $\hat{v}$ is 940 to the nearest 10. You should calculate the value to the nearest 0.1.

(d) (*3 points*) You are given that $\hat{a} = 12,231$

(i) Calculate the credibility factors, Z_0, Z_1, Z_2.

(ii) Explain briefly why, in general, we expect $Z_0 > Z_1 > Z_2$.

(iii) Show that $\hat{\mu} = 452.77$.

(iv) Calculate the Bühlmann-Straub estimate of the projected cumulative claims for AY 2.

(e) (*2 points*) The Bornhuetter-Ferguson method can also be expressed as a credibility estimate. Describe two advantages and one disadvantage of the Bühlmann-Straub approach compared with the Bornhuetter-Ferguson method. You do not need to describe the Bornhuetter-Ferguson method in detail.

6.

(*11 points*) A reinsurance company is analyzing the tail risk of cyber losses. The following table provides the largest 20 values from a sample of 500 cyber losses, in decreasing order.

Rank	**500**	**499**	**498**	**497**	**496**	**495**	**494**	**493**	**492**	**491**
Loss X	21,355	12,495	10,751	8,450	7,548	7,530	6,354	5,668	5,617	5,498
ln(X)	9.969	9.433	9.283	9.042	8.929	8.927	8.757	8.643	8.634	8.612
Rank	**490**	**489**	**488**	**487**	**486**	**485**	**484**	**483**	**482**	**481**
Loss X	5,388	5,326	5,247	5,184	5,092	5,088	4,953	4,668	4,635	4,469
ln(X)	8.592	8.580	8.565	8.553	8.535	8.535	8.508	8.448	8.441	8.405

You are given that the sum of the natural log values of the largest 20 losses is 175.391.

(a) (*2 points*)

(i) Calculate the empirical 99% VaR.

(ii) Calculate the empirical 99% Expected Shortfall (ES).

The data is fitted to the Generalized Pareto Distribution (GPD) with a threshold of $d = 4{,}953$. At this threshold, the maximum likelihood estimates (MLE) of the GPD parameters are $\hat{\xi} = 0.74$ and $\hat{\beta} = 1{,}100$.

(b) (*3.5 points*)

(i) Show that the 99% VaR of the cyber losses using the fitted GPD is 7,000 to the nearest 100. You should calculate the value to the nearest 1.

(ii) Calculate the 99% ES of the cyber losses using the fitted GPD.

(c) (*1 point*) Show that the Hill estimator of $\alpha = \xi^{-1}$, denoted $\hat{\alpha}^H$, is 3.1 to the nearest 0.1. You should calculate the value to the nearest 0.01.

6. **Continued**

(d) (*3.5 points*) The Hill estimator for the tail survival function has the form

$$\hat{S}^{H}(x) = 0.032\left(\frac{x}{d}\right)^{-\hat{\alpha}^{H}}$$, where the threshold is $d = 4{,}953$.

(i) Show that the 99% VaR of the cyber losses using the Hill estimator of the survival function is 7,200 to the nearest 100. You should calculate the value to the nearest 1.

(ii) Calculate the 99% ES of the cyber losses, using the Hill estimator of the survival function.

(e) (*1 point*) State with reasons whether you would recommend using the empirical estimate from part (a)(ii), the MLE GPD estimate from part (b)(ii), or the Hill estimator of the survival function estimate from part (d)(ii) for evaluating the 99% Expected Shortfall.

****END OF EXAMINATION****