SOCIETY OF ACTUARIES

Advanced Long-Term Actuarial Mathematics Exam

Exam ALTAM

Date: Tuesday, April 21, 2026

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has 6 questions numbered 1 through 6 with a total of 60 points. The points for each question are indicated at the beginning of the question.
2. **Question 1 is to be answered in the Excel workbook. For this question only the work in the Excel workbook will be graded.**
3. Questions 2-6 are to be answered in pen in the Yellow Answer Booklet provided. For these questions graders will only look at the work in the Yellow Answer Booklet. Excel may be used for calculations, for referencing tables, or for statistical functions, but any work in the Excel booklet will not be graded.
4. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions provided in this document.

Excel Answer Instructions

1. For Question 1, you should answer directly in the Excel Question worksheet. The question will indicate where to record your answers.
2. You should generally use formulas in Excel rather than entering solutions as hard coded numbers. This will aid graders in assigning appropriate credit for your work.
3. Graders for Excel questions will not have access to any comments or calculations provided in the Yellow Answer Booklet.
4. For Question 1, you may add notes to the Excel Question worksheet if you feel that might help graders. However, these should be entered directly into the Excel Question worksheet. Graders may not be able to read notes entered as comments.
5. When you finish, save your Excel workbook with a filename in the format xxxxx_ALTAM where xxxxx is your candidate number. Your name must not appear in the filename.
6. Record your candidate number in the indicated cell in the Excel Question worksheet.

Pen and Paper Answer Instructions

1. Write your candidate number and the number of the question you are answering at the top of each sheet. Your name must not appear.
2. Start each question on a fresh sheet. You do not need to start each sub-part of a question on a new sheet.
3. Write in pen on the lined side of the answer sheet.
4. The answer should be confined to the question as set.
5. When you are asked to calculate, show all your work including any applicable formulas in the Yellow Answer Booklet.
6. If you use Excel for calculations for pen and paper answers, you should include as much information in the Yellow Answer Booklet as if you had used a calculator, including formulas and intermediate calculations where relevant. Written answers without sufficient support may not receive full or partial credit.
7. When you finish, hand in <u>all</u> your written answer sheets to the Prometric Center staff. Be sure to hand in all your answer sheets because they cannot be accepted later.

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*****BEGINNING OF EXAMINATION****
*****ADVANCED LONG-TERM ACTUARIAL MATHEMATICS*****

Provide the response for Question 1 in the Excel Question worksheet

1.

(*9 points*) An insurance company is analyzing the mortality of its policyholders using the alive-dead model with piecewise-constant forces of mortality between integer ages.

There are *n* lives in the study. For the *j*-th life, let x_j denote the age at purchase of the policy, let t_j denote the time from entry to exit, and let $\delta_j = 1$ if the policy terminated through death, $\delta_j = 0$ otherwise.

For integer age *x*, let w_x denote the total waiting time (central exposed to risk) and let d_x denote the number of deaths observed, between ages *x* and *x*+1.

Assume the lives are independent with respect to their mortality experience.

(a) (*2 points*) Table 1 provides (x_j, t_j, δ_j) for a subset of 5 lives, labelled $j = 1, 2, ..., 5$. Use Table 1 to calculate w_x and d_x for this subset, for $x = 50, 51,$ and 52.

Table 1

j	x_j	t_j	δ_j
1	50.0	1.5	1
2	50.3	1.7	0
3	50.5	0.3	1
4	51.1	0.8	1
5	52.3	3.9	1

(b) (*3 points*) The full data set is used to construct Table 2, which shows w_x and d_x for $x = 50, 51, ..., 59$. Also in Table 2 are columns for the following functions:

$\hat{\mu}_x$ which denotes the maximum likelihood estimate (MLE) of μ_x.

$l(\hat{\mu}_x)$ which denotes the log likelihood function evaluated at the MLE of μ_x.

$SD(\hat{\mu}_x)$ which denotes the estimated standard deviation of μ_x.

Complete Table 2.

1. **Continued**

Table 2					
x	w_x	d_x	$\hat{\mu}_x$	$l(\hat{\mu}_x)$	$SD(\hat{\mu}_x)$
50	98.9	3			
51	97.3	4			
52	97.5	5			
53	97.2	7			
54	96.0	8			
55	96.5	9			
56	95.0	8			
57	94.7	10			
58	94.1	12			
59	94.5	13			

(c) (*3 points*) In Table 3, $\hat{q}_x$ denotes the MLE of q_x, and $SD(\hat{q}_x)$ denotes the estimated standard deviation of $\hat{q}_x$ evaluated using the delta method. Complete Table 3.

Table 3				
x	w_x	d_x	$\hat{q}_x$	$SD(\hat{q}_x)$
50	98.9	3		
51	97.3	4		
52	97.5	5		
53	97.2	7		
54	96.0	8		
55	96.5	9		
56	95.0	8		
57	94.7	10		
58	94.1	12		
59	94.5	13		

(d) (*1 point*) The insurer intends to use the data in Table 3 to construct a life table. The chart on the Excel Sheet shows your estimated values of q_x from part (c) above. **Note that this chart will be automatically generated based on your entries in Table 3.**
Explain why the insurer might modify these estimates of q_x before they are used.

2.

(*10 points*) An insurer offers a fully discrete two-year special joint life insurance policy to a couple aged x and y. You are given the following policy information:

(i) A death benefit of 10,000 is payable at the end of the year of the first death if it occurs within two years.

(ii) A survival benefit of 500 is payable at the end of year 2 if both lives survive.

(iii) A 50% refund of the first premium without interest is payable at the end of year 1 if the policy is surrendered at that time.

(iv) The policy terminates upon the earliest of the first death, surrender, or maturity.

(v) The gross annual premium is G.

The assumptions used for profit testing this policy are:

(i) The pre-contract expense is 100.

(ii) Maintenance expenses of 50 are incurred at the start of each year.

(iii) The commissions are 5% of the gross premium in each year.

(iv) The earned interest rate is 3%.

(v) The hurdle rate is 10%.

(vi) The future lifetimes of (x) and (y) are independent.

(vii) The mortality rates are:

$$q_x = 0.02 \quad q_{x+1} = 0.03 \quad q_y = 0.04 \quad q_{y+1} = 0.05$$

(viii) 10% of the policies in force at the end of the first year surrender.

(ix) The insurer does not hold reserves for this policy.

(a) (*2.5 points*)

(i) Determine Pr_0.

(ii) You are given that $\text{Pr}_1 = 400$. Show that $G = 1100$ to the nearest 100. You should calculate the value to the nearest 0.1.

(iii) Show that $\text{Pr}_2 = -200$ to the nearest 10. You should calculate the value to the nearest 0.1.

2. Continued

(b) (*2.5 points*)

(i) Determine the profit signature, Π.

(ii) Calculate the Net Present Value.

(iii) Calculate the profit margin to the nearest 0.0001.

(c) (*3 points*)

(i) Explain why the insurer would be concerned about the negative emerging profit at time 2.

(ii) The insurer decides to hold a reserve at time 1, denoted ${}_1V$ such that $\text{Pr}_2 = 0$. Calculate ${}_1V$.

(iii) Calculate the revised Net Present Value.

(iv) Explain why the Net Present Value has decreased.

(d) (*2 points*)

(i) Describe two factors that insured couples should take into consideration in deciding whether or not to surrender their policy at the end of the first year.

(ii) Explain how adverse selection might impact the insurer's assumption about mortality in the second year.

3.

(*11 points*) You are using the Standard Sickness-Death multiple state model to price a whole life insurance policy issued to (50), with a waiver of premiums on sickness.

You are given the following policy information and assumptions:

(i) The death benefit of 100,000 is payable immediately on death.
(ii) Premiums are payable at the start of each of the first 20 years, if the policyholder is Healthy.
(iii) Premiums are waived if the policyholder is Sick at the premium payment date.
(iv) Commissions are 45% of premium in the first year and 8% of premiums paid thereafter. Commission is not paid on waived premiums.
(v) All policyholders are Healthy at the issue date.
(vi) Transitions follow the Standard Sickness-Death model given in the Excel worksheet.
(vii) $i = 0.05$
(viii) The Two-Term Woolhouse's Formula is used to calculate annual payment annuity values.

(a) (*3 points*)

(i) Show that ${}_{20}p_{50}^{01} = 0.21$ to the nearest 0.01. You should calculate the value to the nearest 0.0001.

(ii) You are given that ${}_{20}p_{50}^{00} = 0.49695$. Show that $\ddot{a}_{50:\overline{20}|}^{00} = 11.22$ to the nearest 0.01. You should calculate the value to the nearest 0.0001.

(b) (*2.5 points*) Let L denote the gross loss-at-issue random variable, and let G denote the gross annual premium. You are given that $E[L] = -100$.

Show that $G = 3{,}340$ to the nearest 10. You should calculate the value to the nearest 0.1.

3. Continued

You are given the following additional information:

- ${}_tV^{(j)}$ denotes the gross premium policy value at time t for a policy in State j at time t.
- For integer t, ${}_tV^{(j)}$ denotes the value immediately before the time t premium is due to be paid, and ${}_{t^+}V^{(j)}$ denotes the value immediately after the time t premium is due to be paid.
- $\ddot{a}^{00}_{60:\overline{10}|} = 6.9073 \qquad {}_{10}V^{(1)} = 49{,}334$
- $\mu^{01}_{60} = 0.014086 \quad \mu^{02}_{60} = 0.014518 \quad \mu^{10}_{60} = 0.003443 \quad \mu^{12}_{60} = 0.02325$

(c) (*3 points*)

(i) Show that ${}_{10}V^{(0)} = 25{,}000$ to the nearest 100. You should calculate the value to the nearest 1.

(ii) Show that ${}_{10^+}V^{(0)} = 28{,}100$ to the nearest 100. You should calculate the value to the nearest 1.

(iii) Write down the value of ${}_{10^+}V^{(1)}$.

(d) (*2.5 points*)

(i) Show that $\lim_{t\to 10^+} \frac{d}{dt}\,{}_tV^{(0)} = 28$ to the nearest 1. You should calculate the value to the nearest 0.001.

(ii) Calculate ${}_{10.2}V^{(0)}$ using Euler's method with a step size of $h = 0.2$.

4.

(*10 points*) A company has a hybrid pension plan combining a Final Average Salary Defined Benefit (DB) plan with a Defined Contribution (DC) top-up.

You are given the following plan information and valuation assumptions for a valuation at 1/1/2026.

(i) The DB accrual rate is 1.6%.

(ii) Pensions are paid monthly in advance.

(iii) Final average salary is defined as the average salary paid in the three years before retirement.

(iv) Employees have the option to contribute to a separate DC account throughout their working lives.

(v) Salaries increase at the end of each year by 2%.

(vi) Retirements before age 65 are assumed to occur midway through the year of age.

(vii) $i = 0.045$

(viii) $\ddot{a}_{64.5}^{(12)} = 13.882$ and $\ddot{a}_{65}^{(12)} = 13.723$ at $i = 0.045$

(ix) The Projected Unit Credit method is used for valuation and funding.

The following is an excerpt from the valuation service table. There are no retirements before age 64.

x	l_x	r_x
23	73,804	0
24	66,696	0
⋮	⋮	⋮
63	⋮	0
64	⋮	865
65 exact	8,203	8,203

A new employee, Jeff, joins the plan on the valuation date at age 23, with a starting salary of 80,000.

4. Continued

(a) (*4 points*)

(i) (*1 point*) Show that the projected final average salary (FAS) at age 64.5 is 175,000 to the nearest 1000. You should calculate the value to the nearest 1.

(ii) (*1 point*) Show that the projected final average salary (FAS) at age 65 is 177,000 to the nearest 1000. You should calculate the value to the nearest 1.

(iii) (*2 points*) Calculate the normal contribution rate for Jeff's DB plan retirement benefits at the valuation date.

(b) (*2 points*) Show that Jeff's projected replacement rate from the DB plan if he retires at age 65 is 0.66 to the nearest 0.01. You should calculate the value to the nearest 0.001.

(c) (*2.5 points*) Jeff plans to contribute 1% of his annual salary to the DC fund, and to retire at age 65. You are given the following assumptions:

(i) Jeff's contributions are made at each year end, with the last payment at age 64.

(ii) The DC account earns 8% interest per year.

(iii) At retirement, Jeff plans to annuitize the DC account value. The annuitization assumptions are the same as for the DB valuation.

Calculate Jeff's total projected replacement rate at age 65.

(d) (*1.5 points*) Describe one advantage and one disadvantage of this hybrid plan from the employee's perspective, compared with a DB plan with a higher accrual rate and an annual employee contribution of 1% of salary.

5.

(*10 points*) You are developing a Type A last survivor universal life policy of 1,000,000 on a couple, Xiaobai and Yvette. At the issue date Xiaobai is 51 and Yvette is 61. You are given:

(i) The death benefit is paid at the end of the year of the second death.
(ii) Premiums are payable at the start of each year.
(iii) Account values are calculated annually; AV_t denotes the account value at *t*.
(iv) Xiaobai and Yvette have independent future lifetimes.
(v) The cost of insurance (COI) is based on 100% of the mortality in the Standard Ultimate Survival Model (SUSM).
(vi) The interest rate for calculating the COI is $i_q = 0.03$.
(vii) Initial expense charges are 200 plus 40% of the first premium.
(viii) Renewal expense charges for the second and subsequent policy years are 50 plus 10% of the premium.

(a) (*1 point*) Show that $1000 \times q_{\overline{51:61}} = 0.005$ to the nearest 0.001. You should calculate the value to the nearest 0.00001.

(b) (*3 points*) The couple pays a premium of 14,400 at the start of the first year. The crediting rate in the first year is 5%.

(i) Calculate AV_1 if both insureds remain alive at the end of the first year.

(ii) State with reasons whether AV_1 would be larger, smaller, or the same, if only Yvette survives to the end of the first year.

(c) (*2.5 points*) You are given that $AV_6 = 92,000$, a premium of 14,400 is paid at $t = 6$, and the crediting rate in the 7th year is 5%.

(i) Calculate AV_7 if both lives are alive at $t = 6$.

(ii) Calculate AV_7 if only Xiaobai is alive at $t = 6$.

5. **Continued**

(d) (*3.5 points*) The policy has a no lapse guarantee. At the end of year 10, based on this policy's premium history, it cannot lapse before the end of year 20.
The reserve for the no lapse guarantee is based on the following assumptions:

- Mortality follows the SUSM.
- $i = 0.05$

You are given that $AV_{10} = 127,000$.

(i) Show that $A^{\frac{1}{}}_{\overline{61:71:10|}} = 0.008$ to the nearest 0.001. You should calculate the value to the nearest 0.0001.

(ii) Determine the reserve for the no lapse guarantee at the end of year 10 if both lives are alive at time 10.

(iii) Determine the reserve for the no lapse guarantee at the end of year 10 if only Yvette is alive at time 10.

6.

(*10 points*) An insurer issues a 10-year equity-linked contract to (70). The single premium is 100,000. You are given the following information and assumptions.

(i) There is a Guaranteed Minimum Maturity Benefit (GMMB) of 100% of the premium.

(ii) An initial expense charge of 500 is deducted at the start of the contract.

(iii) Management charges of 2% of the policyholder's account value are deducted at the start of each year of the contract including the first (after the initial expense deduction).

(iv) The underlying asset values of the policyholder's account follow a lognormal process with parameters $\mu = 0.05$ and $\sigma = 0.25$.

(v) Mortality is assumed to follow the Standard Ultimate Survival Model (SUSM).

(vi) There are no lapses.

(vii) The risk free rate of interest is $r = 0.04$ per year, compounded continuously.

(a) (*3 points*) Show that the cost of the GMMB is 13,540 to the nearest 10. You should calculate the cost to the nearest 1.

(b) (*1.5 points*)

(i) Determine the value of the stock part of the initial delta hedge portfolio (that is, the value of the investment in the underlying asset).

(ii) Explain briefly why the GMMB hedge portfolio takes a short position in the underlying assets.

(c) (*2.5 points*) Calculate the probability that the GMMB matures in-the-money (that is, the probability, under the real world measure, that the contract is in force at maturity, and that the policyholder's fund value at that time is less than the GMMB).

6. Continued

The contract has an additional Guaranteed Minimum Income Benefit (GMIB). At time 10 the policyholder may annuitize the greater of the policyholder's fund value at maturity, and the policy's Benefit Base at maturity. You are given the following additional information.

- The Benefit Base is defined as 100% of the premium with interest at 4% per year.
- The guaranteed annuitization rate at age 80 is 10.5%.
- The annuity benefit is paid annually at the start of each year.

(d) (*2 points*)

(i) Calculate the guaranteed minimum amount of annual annuity income payable from age 80, based on the information available at the policy inception.

(ii) The insurer calculated the guaranteed annuitization rate using the SUSM at a rate of interest of j per year effective. State with reasons whether j is greater than or less than 5%.

(e) (*1 point*) Explain briefly why the cost to the insurer of the GMIB at maturity could be less than the Benefit Base at maturity, even if the GMIB is exercised.

****END OF EXAMINATION****